Errata

Classical Mathematical Logic

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p. 41

Theorem 6.e is incorrect. Either of the following are correct and each is sufficient for any proof later in the text that depended on the erroneous version.

(1) If Γ is a complete theory, then for every A, either $A \in \Gamma$ or $\neg A \in \Gamma$.

(2) If Γ is a theory, then Γ is complete iff for every A, either $A \in \Gamma$ or $\neg A \in \Gamma$.

p. 47

The displayed equivalence in Theorem 15 should read:

$v(A \wedge B) = T$ iff $(A \wedge B) \in \Gamma$
iff $A \in \Gamma$ and $B \in \Gamma$ using **axioms 5**, **6** and **7**
iff $v(A) = T$ and $v(B) = T$

pp. 101 and 102

A stronger version of Theorem 5 is needed later. The proof is a minor modification of the one given in the text.

Theorem 5 Let $y_1, \ldots, y_n$ be a list of all variables free in A. Let $t_1, \ldots, t_n$ be any terms such that t_i is free for y_i in A. Let σ, τ be any assignments of references such that for each i, $\sigma(y_i) = \tau(t_i)$. Then

$$\sigma \models A(y_1, \ldots, y_n) \text{ iff } \tau \models A(t_1/y_1, \ldots, t_n/y_n).$$

Proof We proceed by induction on the length of A. If A is atomic, this is the extensionality restriction. Suppose now that the theorem is true for all wffs of length $\leq m$ and A is of length $m + 1$. I will leave the cases when A is $B \wedge C$, $B \vee C$, $B \rightarrow C$, or $\neg B$ to you. Suppose that A is $\forall x\, B$. Then:

$\sigma \models \forall x\, B(x, y_1, \ldots, y_n)$ iff for every γ that differs from σ at most in what it assigns x, $\gamma \models B(x, y_1, \ldots, y_n)$

iff for every δ that differs from τ at most in what it assigns x, $\delta \models B(x, t_1/y_1, \ldots, t_n/y_n)$
(by induction, since x does not appear in any of $t_1, \ldots, t_n$ as these are free for $y_1, \ldots, y_n$ in A, and so $\delta(t_i) = \tau(t_i) = \sigma(y_i)$)

iff $\tau \models \forall x\, B(x, t_1/y_1, \ldots, t_n/y_n)$

The case when A is $\exists x\, B$ is done similarly. ■

The proof of Theorem 3 on p. 101 is then considerably simplified by invoking this Theorem 5:

(a) By way of contradiction, suppose some $\sigma, \sigma \nvDash A(y/x)$. Let τ be such that for all z other than y, $\sigma(z) = \tau(z)$ and $\sigma(y) = \tau(x)$ Then by Theorem 5, $\tau \vDash A(x)$ iff $\sigma \vDash A(y/x)$. Hence, $\tau \nvDash A(x)$. Part (b) is proved similarly.

p. 118
The proof of Lemma 3.d is as for Theorem VI.5 on p. 102 given directly above.

p. 119
line 17 from bottom, read "t_i is x" for "$t_i = x$" and "t_i is a_i" for "$t_i = a_i$"
line 14 from the bottom, read "closed wff B" for "wff B".

p. 120
Add an exercise:
7. Show that for every model M there is a model N which satisfies the predicate logic criterion of identity such that for every closed wff A, $\mathsf{M} \vDash A$ iff $\mathsf{N} \vDash A$.

p. 171
line 15, read "Axiom 3" for "Axiom 2".

p. 172
line 3 should read:

Hence, by *modus ponens*, we have $\vdash \forall x\, (\forall \ldots)_1\, (\forall y\, A(x) \rightarrow A(x))$, as desired.

p. 173
In the definition of Σ_{n+1} each appearance of "Σ" should be replace by "Σ_n".

p. 173
line 6 from the bottom delete "hence by Lemma 1" and replace with:

> Σ is also a theory: if $\Sigma \vdash A$ and if $A \notin \Sigma$, then by construction, $\neg A \in \Sigma$, so Σ would be inconsistent, so $A \in \Sigma$.

p. 174
in the definition of the model it should read:

$\sigma \vDash \mathsf{P}_i^n(t_1, \ldots, t_n)$ iff $P_i^n(\sigma(t_1), \ldots, \sigma(t_n)) \in \Sigma$

p. 174
line 15 from the bottom
read "by universal instantiation, $\mathsf{M} \vDash B(v_i/x)$" for "if $\sigma(x) = v_i$, then $\sigma \vDash B(x)$"

p. 182
line 5 from the bottom read "$\exists$ " for "ε" .

p. 423
In Theorem 1 delete part (f) which is wrong and not needed.

p. 424
line 9 from the bottom read "Σ" for "Γ".

p. 424
line 7 from the bottom to the end of the page should read:

$\mathsf{U} = \{c_i$: for some $x, \exists x\,(x \equiv c_i) \in \Sigma\} \cup \{v_i$: for some $x, \exists x\,(x \equiv v_i) \in \Sigma\}$

Assignments of references:

For every σ and every x, $\sigma(x)$ is defined, and the collection of such σ is complete.

For every atomic name d:

$\sigma(d)\downarrow$ iff $d \in \mathsf{U}$. If $\sigma(d)$ is defined, then $\sigma(d) = d$.

Evaluation of the equality predicate:

$v_\sigma \vDash t \equiv u$ iff $\sigma(t) = c$ and $\sigma(u) = d$, and $(c \equiv d) \in \Sigma$
or both are undefined and $(t \equiv u) \in \Sigma$

Valuations of atomic wffs other than the equality predicate:

Given $A(t_1, \ldots, t_n)$ and σ, let $y_1, \ldots, y_n$ be a list of all the variables appearing in A, and let $\sigma(y_i) = d_i$. Let $A(t_1, \ldots, t_n)[d_i | y_i]$ denote A with each y_i replaced by d_i. Then:

$v_\sigma \vDash A(t_1, \ldots, t_n)$ iff $A(t_1, \ldots, t_n)[d_i | y_i] \in \Sigma$.

p. 429
The second and third inferences at the top of the page are invalid, not valid.

p. 431–432
The following conditions replace comparable parts or are added to the text.

Extensionality condition for atomic applications of terms For any atomic terms $t_1, \ldots, t_n$ and $u_1, \ldots, u_n$, if for all i, $\sigma(t_i)\downarrow = \tau(u_i)\downarrow$, then:
either both $\sigma(f(t_1, \ldots, t_n))$ and $\tau(f(u_1, \ldots, u_n))$ are undefined, or both are defined and are the same object.

Applications of functions extended to all terms

Terms of depth 0: These are given.

Terms of depth 1: These are given satisfying non-referring as default and the extensionality condition for atomic applications.

Terms of depth $m + 1$ for $m > 0$:

Applications of functions for terms of depth $\leq m$ are given.

For $f(t_1, \ldots, t_n)$ a term of depth $m + 1$:

If for some i, $\sigma(t_i)\not\downarrow$, then $\sigma(f(t_1, \ldots, t_n))\not\downarrow$.

If for all i, $\sigma(t_i)\downarrow$, let $z_1, \ldots, z_n$ be the first variables not appearing in $f(t_1, \ldots, t_n)$. Let τ be the assignment of references that differs from σ only in that for all i, $\tau(z_i) = \sigma(t_i)$. Then: $\sigma(f(t_1, \ldots, t_n)) \approx \tau(f(z_1, \ldots, z_n))$.

The extensionality of atomic predications If A is an atomic wff, and σ and τ are any assignment of references, and $t_1, \ldots, t_n$ are all the atomic terms in A, and $u_1, \ldots, u_n$ are any terms such that for each i either $\sigma(t_i) = \tau(u_i)$ or $\vDash u_i \equiv t_i$, then $v_\sigma \vDash A(t_1, \ldots, t_n)$ iff $v_\tau \vDash A(u_1 | t_1, \ldots, u_n | t_n)$.

Evaluation of the universal quantifier with partial functions

$v_\sigma \vDash \forall x\, A(x)$ iff for every term t that is either x or contains no variable that appears in $A(x)$, for every τ that differs from σ at most in what it assigns x, $v_\tau \vDash A(t | x)$