Logic, Language, and the World

Volume 3

Reasoning about the World as Process

Richard L. Epstein

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For more information visit our website:

www.AdvancedReasoningForum.org

Or contact us:

Advanced Reasoning Forum
P. O. Box 635
Socorro, NM 87801 USA
rle@AdvancedReasoningForum.org

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Preface

It is hard for us to conceive of the world as process, flux, the flow of all, with no or only a quite secondary idea of there being persisting individual things. In *Language and the World: Essays New and Old*, I've tried to make it possible for you to enter that way of encountering the world, with my essays and essays by linguists and anthropologists who have described people who talk and live with that conception. That is important and useful background, but not, I hope, essential, for I have tried to set out the basic idea of the world as flow in the first three chapters.

That conception is difficult to grasp for speakers of English, German, and Romance languages. And even grasping it, a little, it is hard for us to imagine how to use it. If it is not mysticism but a change of grammar, then it should be possible to stipulate that grammar more or less clearly. Using that, we can try to set out rules for how to reason well according to that conception and grammar. That is what I hope to do here.

This is an attempt, a first attempt as far as I know, to formulate a formal logic that is not tied to our own European languages, to step out of our language conceptions and habits. To do so, I have drawn on many ideas of Western formal logic, as set out in the other volumes in this series. That has led to this work relying to some extent on the idea of individual thing, though I hope not essentially, as I discuss in many places in the text. In the end, then, this work is a bridge, a chance for us as speakers of languages that focus primarily on the world as made up of things to begin to see the richness and complexity of the way of encountering the world as the flow of all, the one and not the many. The contrasts, constant and often unsettling, will, I hope, lead us to better understand how we encounter and reason about the world as made up of things.

The World and Language

1 Flux and Continuity

We have two great certainties in our lives. We are certain that everything changes, that nothing stays the same, that all is flux. This is our first experience of the world, our earliest perception. And later we have the certainty that some things persist, some things are the same. The ball we played with yesterday is the same we have in our hand today, this house is the same house. The certainty that things persist in time is overlaid on our certainty that all is flux, for we believe in the continuity of the ball, the house, though we know that they are not the same: the ball has been scuffed, the house has been painted. Our certainty that things persist is overlaid and contradicted by our understanding that all is flux, but we hold to it nonetheless. We construct our certainty in the persistence of things from our experience of the flux, learning quite young how to establish correlations, equivalences, sameness of things. This is the same ball, the same house, where we learn that what we mean by "same" depends on the kind of thing.

Nowhere is this more evident than in our certainties about ourselves. Nothing is more certain to us than that we change: we cut a finger, we get a haircut, we cannot see as well as we once did, we have a backache, we limp, we squint when we could see so well before. And we believe differently now: we see our first love so differently, we remember and understand the job we left, the friends we still have from former times in a very different way, all summed up in "If I knew then what I know now . . .". We know we change, but our greatest certainty, beyond the certainty that we change, beyond the certainty that all is flux, beyond the certainty that things persist, is that there is an "I", a me, a single person that persists, that unifies the changes in our physique and the changes in our perceptions. This "I" we are certain of from the time we first become able to formulate the idea when we are very young until we lose the ability to formulate the idea when we are very old.

We do not know well how to reason about things and ourselves in flux. We build our reasoning on the certainty of the persistence of things in time. We do so in the extreme when we use predicate logic with tenseless propositions, true in a time-less way, with names that pick out things that are just that thing forever, though the thing does change. We do so less when we take time into account in our reasoning, when we index propositions with time or periods of time relative to each other. We begin, slowly, to take account of flux and change. We do so more when we index not only propositions but predicates with time.

There is, or sadly I should say was Juney. She was a black and white border collie. The first day I met her my neighbor, an elderly lady, had just got her from a relative who didn't want the dog and who had kept Juney tied up. My neighbor asked me to tie Juney in my yard while she had some relatives over because Juney was too active, a young dog, almost a puppy, and wild. I tied her up in my yard and

went to pet her. She bit at me, afraid, unhappy to be tied up again. That was Juney on that day. And the Juney of each day thereafter, slowly learning to trust me, going for a walk with me every day. And the Juney of two years later, who would pass up her food to go for a walk with me, who would not go into my neighbor's home at night but would lie outside my kitchen window looking up at me. There was one Juney, I say, knowing that there was a unity, a single Juney, that somehow, some way, unifies each of those daily Juneys. The closest we can come to recognizing in our reasoning both the certainty of the unity and the certainty of the flux is to use a single name, "Juney". But what and how that picks out a thing, has a reference, stands for something, we need to consider. And how we can talk of the flux, of change and process, of mass, that is what we will most consider here.

2 Analogies

Much of how we "deal with" or "make sense of" the world is by implicit comparing.[1] We say "this" is like "that" and use the comparison in not only our reasoning but in how we move and act in the world.

Yet that way of describing what we do is already a great abstraction. It assumes there is some *thing* that is the "this" or the "that". The more basic issue is how we construct things out of our experience. The general method of parceling out experience into bits seems to be the heart of how we not only "see" the world, but how we communicate, the basis of our language.

We draw analogies constantly to make sense of our experience. Then those analogies become crystallized in our language. For example, some time long ago some person and then some people began to view water flowing from a small river into a larger one as analogous to (similar to) a vassal paying gold or animals or labor to his king or overlord, and used the same word for both the small river and the vassal: "tributary". Now when we use that word it has the two distinct meanings that are joined by the analogy we no longer think about but which guides us. Or we use the word "rotten" for ice that has frozen, thawed, refrozen, and now will no longer support weight, accepting without thought an analogy to rotten fruits and rotten meat, which are quite different from each other, and from rotten people and rotten luck, which are more different still.

Each of us makes analogies constantly. Many of them are idiosyncratic. But many of them are guided by our language. Every speaker of English, wishing to or not, makes an analogy between attempting and choosing a new experience when using the word "to try", whereas a speaker of Portuguese does not have that analogy ready-made, using two distinct words "tentar" and "experimentar". Every speaker of Portuguese, wishing to or not, accepts waiting, hoping, and expecting as analogous by using the one word "esperar", whereas a speaker of English finds them dissimilar in having to choose which of "wait", "hope", or "expect" to use. Often what we call various senses of a word amount to codifications of analogies, or rather, looser analogies, since every use of a word draws some analogy with how we have used it in other contexts.[2]

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[1] In science and mathematics we try to make the comparisons explicit, as I discuss in "Models and Theories", "Generalizing", and "Mathematics as the Art of Abstraction".

[2] Gary Witherspoon in *Language and Art in the Navajo Universe*, says:

> Most words and linguistic expressions label concepts and conceptual classes, and these concepts and classes are all based on analogical propositions; that is, they lump together under one label things and events that are distinct and separate but which do in some ways resemble one another. No two oak trees—even the two of the same variety in the same forest—ever resemble each other in exact detail, and even if their resemblance were exact, their position in space and their material constitution would be different. But because of their resemblance, we call them both oak trees. Earth Woman, human

Some analogies codified in our language are given not by a choice of word but by grammar. In English we see the act of throwing and the act of chasing as similar, in that both require a subject and an object. We see the act of kissing as somewhat but not entirely similar, in that it can have a subject and object, but need not, for it can have a conjunctive subject as in "Dick and Zoe kissed". We see the act of cursing as different still, in that it has to have a subject but may or may not have an object, and then the subject need not be conjunctive. We see the act of dying as quite different, not analogous at all, since it can have no object. And we see the act of raining as very different, for it has no subject. These comparisons, analogies, and distinctions are built into our language; we cannot escape them. There is no reason to think that another language will make the same analogies and disanalogies.

At least we cannot escape them in the daily use of our language. We can try to escape them by recognizing them explicitly rather than just using them implicitly, comparing them to analogies and disanalogies in other languages, and reflecting on our and others' use of language and seeing the world. That is what we will do here.

> mothers, the sheep herd, corn fields, and the mountain soil bundle are all called *shimá*. It is not adequate or accurate, however, to say simply that some are mothers and some are not mothers but are metaphorical extensions of the concepts of motherhood. They are all mothers, but each one is a mother in a way that in part resembles the others but in part is distinct from the others. p. 93

His point, as he explains later, is that our calling these uses of the word *shimá* metaphorical on the notion of biological mother is to impose on the Navajo our categories of what is significant, for they do not take any of these meanings as metaphorical.

3 The World as Process

The world is made up of things: rocks, tables, dogs, people, stars. Nothing is more fundamental in our experience than this perception. It's in our language with words for all these and many more.

We know of process and change, too. But we know of them only through things. For example, suppose I show you an apple. It's round, red, shiny. I take a bite of it. It's changed—no longer round, no longer red and shiny where I bit into it. I take another bite. The apple has changed some more. I take another bite, and another, and the apple has changed a lot. I give the core to my donkey. The apple is all gone.

The apple changed. But is that the apple I started with? If one apple changed, it wasn't what I first showed you, it wasn't what I bit into the second time, it wasn't the core. It must have been something beyond all those, somehow beyond any particular time, something that persists through all "its" changes. Talking of change we find ourselves talking about things beyond any particular time.

Change, we feel, is not real like things are real, like rocks, tables, dogs, people, stars, the sun. The sun? Everything we know about that fiery ball tells us that the sun is a process: nothing endures in it, not shape, not form, not even molecules—only the process. A rock, too, is process, changing, never stable, though we don't notice the changes. The difference isn't that the sun is a process and the rock is a thing; the difference is the scale of time over which we note "changes".

Our focus in our language is on the world as made up of things, on stability in the flow of our experience. Still, we have some sense in our lives of flow, of flux, of change, of process. And we have some hints of that in our language.

Suppose you're in my living room with me, and I look out the window and say,

It's raining.

Yes, that's true. But what's raining? There's no "it": the weather isn't raining. The weather is rainy; the weather doesn't do anything. The word "it" is a dummy, there because in English every verb requires a subject. I could have said just,

Raining.

You would have understood me. It's clear I'm talking about now, which is all the "is" in the original sentence tells us. And it's clear I'm talking about there, outside the window, though in English we don't require any word or phrase to mark that.

On a winter day I might say "Snowing", and you'd understand me. That's complete, clearly true or false, though it doesn't look like a sentence in English. Or I could say, "Sun-ing" or "Breeze-ing", which are odd, but once you've got the hang of my talking this way, you'd understand me.

If we were at my friend's apartment in the city, I might look out the window and say,

Running.

You'd understand me. It sounds odd because I haven't said who or what is running. That seems essential when we talk English because verbs are descriptions of what's happening to or because of a thing. Yet running is running, whether it's one person, a dog chasing a cat, or lots of people in a marathon. I don't describe all when I say "Running", but we never describe all. What I've said is true or false, enough to communicate.

Looking out my window at the patio at home I could say "Barking" and you'd understand me. On another day looking at my dogs I could say, "Sleeping". These are process words, and used this way they begin to become part of a way to describe process without a focus on things.

After a rain, as I look out at the patio I might say, "Mud". Mud isn't a thing. We don't say "There are three muds out there." We say, "There's some mud" because mud is a mass. Water, gold, snow are masses, too. We know they're part of what the world is made up of, different from things. Every part of mud is mud, while there's no part of an apple that is an apple. Processes are like that, too. Every part of raining is raining—there's no smallest part of raining, for a single drop of water is not raining.

Starting to see the world as process-mass, I look out the window and say, "Dog-ing". You'd understand, though it seems incomplete. One dog or many dogs? What's the dog doing? We need a verb and an indication of singular or plural when we talk in English. Yet if I say, "There's a dog", the verb is just "is". The dog is there, it exists there, that's all. "Dog-ing", understood as about there and now, does that as well, though it doesn't say whether there's one or many, whether alive or dead, whether big or small. Much is left out, but much is left out of our description "There's a dog."

I could turn, and looking around the room say, "Table-ing". You'd understand. An odd way to talk, but true. Or pointing to the next room I could say, "Woman-ing". Odd, too, incomplete, but true. We are beginning to see the world as made up of processes.

Processes? To say that is to slip back into thing-talk. This process, that process, one process, two processes, a fast process, a blue process. No. To see process in the world there are not processes, just process. The world is flux. Words like "raining", "sun-ing", "running", "dog-ing", "mud-ing" describe the flux of all at a time and place. They don't pick out separate parts of the flux any more than "Pacific Ocean" and "Baltic Sea" pick out parts separate and distinct from the water that covers the earth.

To talk of the world as process-mass we can borrow and modify some words from English like "raining", "sun-ing", "running,", "dog-ing", "mud-ing", "woman-ing". We add "-ing" to remind us of our new way of talking, of seeing. When we say what time and place these *base mass-process words* are meant to describe, as

in "Raining (now, here)", we have a "sentence" that is true or false. Looking out the window, if I say "Cat-ing (now, there)", that's false. Given any word *t* that describes a time and any word *l* that describes a location, "Raining (*t*, *l*)" is true or false, "Cat-ing (*t*, *l*)" is true or false, "Woman-ing (*t*, *l*)" is true or false, "Mud-ing (*t*, *l*)" is true or false.

We can describe more fully. Seeing a rabbit running, I can say,

(Rabbit-ing + Running) (now, there)

The flow of all can be described by both those base words in a joined way, as when we say of someone in English that she is a good student-athlete. And that's equivalent to "(Running + Rabbit-ing) (now, there)".

I could point to my patio and say "Brown-ing" and that would be true, for my old brown dog Birta is there. "Brown-ing" as much as "Dog-ing" describes in the flow of all. There is no distinction between what we call adverbs and adjectives because there are no nouns and no verbs, no words for things and what is done by or to them. There are only base words meant to describe the flux at times and locations.

We can describe more fully by saying,

Not–Raining (here, now)

Coyote-ing (yesterday, there) *or* Dog-ing (yesterday, there)

(Rabbit-ing + Running) (there, now) *and*
(Dog-ing + Chasing) (there, now)

All the ways we join sentences in English with the connectives "not", "or", "and", "if . . . then . . ." we can use in talking of the world as process, for those require only that the sentences are true or false, not that they are about things.

In English we get tongue-tied trying to talk of sameness and difference. Is (are?) the apple then and the apple now the same? How can two things be the same? Sameness and difference are very different in the world as process. A visitor to my ranch saw a couple dogs in the corral yesterday. She's standing next to me today and wonders whether those were the same as the dogs that are here in front of us. Is it the same dog-ing? Is the dog-ing then and there the same as the dog-ing here and now? We can formulate that question in mass-process talk
by asking whether the following is true:

Dog-ing (yesterday, corral) ≈ Dog-ing (here, now)

The symbol " ≈ " is not identity of things but similarity, indicating equivalent descriptions, just as we say that my dog Birta today is the same dog as the one I petted yesterday though she's lost some hair, eaten a meal, ...

We can assert similarity without talk of time and location, too:

(Canine-ing + Domestic-ing) ≈ Dog-ing

This is not a universal statement that at any place and time "(Canine-ing + Domestic-ing)" describes the same as "Dog-ing". Rather , the concept, the category, the genus if you will of "Canine-ing + Domestic-ing" is the same (similar to) that of "Dog-ing".

In English we talk of some mud, this mud, that mud, parts of mud. We also say that genus dog is part of genus animal. Basic to our talk of masses is the idea of a part of a mass. But to say "part of" seems wrong, for that brings with it from English the idea of substance. We can have a part of a thing or a part of a mass, but it would be odd to talk of a part of running, even though running is treated as a mass word in English. It would be better to speak of subordination. Abbreviating "subordinate to" as "sub", we'd have that the following are true:

Dog-ing sub Canine-ing

Dig-ing sub Working

And the following are false:

Cat-ing sub Dog-ing

Barking sub Meowing

We have a simple grammar: base words; conjunctions of base words; base words of specific times and places; propositional connectives; a subordination relation; and a similarity relation. This is enough for us to develop a formal logic to guide us in reasoning about the world as process.

In *Language and the World: Essays New and Old* I and others investigate this different way of encountering the world and show how it leads to differences in how we live. That is important not only as motivation for developing a formal logic of reasoning about the world as process but also as a guide for how to think of the world as process. But here it is enough that you have some idea of what kind of metaphysics will be the basis of our formal work.

A Base Logic for Reasoning about the World as Process-Mass

4 Logic and the World as Process

Our goal is to have a language and guide to reason about the world as process-mass. We want to investigate our conception of the world as the flow of all without confusing it with a mixture of the view of the world as made up of things.

For reasoning we need some notion of proposition, what is or can be construed as true or false. In this series of volumes, as in my other work, I assume that utterances or written inscriptions are what are true or false.[1] Does that involve us in talk of things? Aren't inscriptions and utterances things? Isn't a sentence a thing?

Out of the stream of talk we identify parts. No two utterances of "dog" are identical, just as no two bits of mud are. How and why we identify them I cannot say. But we do. That does not make them into things any more than talking of the mud here and the mud there make "those" into things. Whether as specific utterances or types, a part of the flow of speech and writing need not be construed as a thing.

There are many ways we impose or recognize a dichotomy of sentences into those that are correct to proceed on as the basis for good reasoning and those that are not. Each can be and has been used as a true-false division.[2] What I have assumed in the previous volumes of this series is that a proposition is true if it is a good/accurate/correct description of (some part of) the world. That leaves open what is meant by "good" or "accurate" or "correct". It leaves open what we mean by "the world" and a "part of the world". But it has been enough to guide us in developing logics in this series. It doesn't depend on a notion of thing.

But consider:

(1) Every bachelor is an unmarried man.

This is true, but not because it is a good/accurate/correct description of (some part of) the world. The sentence sets out or recognizes part of how we agree to use the word "bachelor", where the agreement need not be explicit.[3] In predicate logic we adopt a formal version of (1) as a meaning axiom, treating it as true in a model but not as a description of the world.[4] With it we constrain how we shall use those words.

Some disagree. They maintain that (1) is true because that's the way the world is. The contrast is clearer if we consider a Latin example:

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[1] The view of propositions as abstract objects is not compatible with seeing the world as the flux of all. But even if absrract propositions exist and are fundamental, utterances and inscriptions can and must stand in for them in our reasoning together, perhaps as representatives or expressions of them, as I explain in "Truth and Reasoning".

[2] See my "Truth and Reasoning". Arne Naess in *"Truth" as Conceived of by Those Who Are Not Professional Philosophers* shows that ordinary folks differ in their ideas of truth in the same way that philosophers do, and no one conception merits being called the common notion.

[3] Much of truth can be understood as based on such agreements, as I discuss in the concluding chapter of *Propositional Logics.*

[4] See *Predicate Logic* or Volume 0: *An Introduction to Formal Logic.*

(2) (a) Homo est animalum.

In English, this translates word-for-word as[5]:

(b) Man is animal.

Many medievals and all modern logicians choose to understand (2) as:

(3) Every man is an animal.

This, they say, is true just as (1) is because it correctly describes the world. That forces a thing-view reading of (2), yet (2) would be true were all men and all animals annihilated. The issue of whether "every" includes "and there exists" is irrelevant. If (2) is true because it correctly describes the world, it is about the relation between genus and species, not the things that are in genus and species. That, indeed, is how medieval realists construed (2), taking genus and species to be abstract things. Nominalists since medieval times say that (2) is true because that's how we do or should use the words. That does not make all truth into just correct use of words: "Socrates was a philosopher" is not about how to use "Socrates".[6]

The reading of (2) as (3) is not possible in the view of the world as process. Nor in the view of the world as process can we understand (2) in terms of abstract things. But we can understand (2a) as true because it correctly tells us how we do or should use the mass-terms "homo" and "animalum". For our work here we will adopt the view of truth as correct description or as correct use of words, seeing those as complementary.

Proposition A *proposition* is a written or uttered piece of language used in such a way that it is true or false, but not both.

A proposition is true if it is a good/accurate/correct description of the world or if it codifies a good/accurate/correct use of words.

A proposition is false if it is not true.

With this assumption, we can define the notion of inference.

Inference An *inference* is a collection of two or more propositions, one of which is designated the *conclusion* and the others the *premises*, that is intended by the person who sets it out as either showing that the conclusion follows from the premises or investigating whether that is the case.

An inference is *valid* means that there is no way the premises could be true and the conclusion false.

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[5] I use the traditional translation rather than the gender-neutral "Human is animal" in order to facilitate comparisons with other expositions and discussions of medieval logic.

[6] Chinese is a mass-process language (see "Nouns and Verbs"). In ancient pre-Han Chinese the focus in analysis of reasoning was on "rectification of names", which seems to include what I have called here "the correct use of words". See Chad Hansen's *Language and Logic in Ancient China*.

An inference is *strong* means that there is a way the premises could be true and the conclusion false, but all such ways are unlikely. An invalid inference that is not strong is *weak*.

If an inference is valid or strong, then the conclusion *follows from* the premises; the conclusion is a *consequence* of the premises.

I'll leave to you to puzzle out whether these definitions depend on a notion of thing. I think they don't.

In this series of books we are focussing on the formal aspects of reasoning.[7]

Formal logic *Formal logic* is (i) the analysis of inferences for validity in terms of the structure of the propositions appearing in the inference, and (ii) the analysis of propositions for truth in terms of their structure.

We now make explicit that we will focus on the view of the world as process.

The World as Process and Propositions The world is process, a flow of all. The propositions in which we are interested are about the world as process.

If this sounds unsatisfyingly vague, compare it to the assumption on which predicate logic is based:

Things, the World, and Propositions
The world is made up at least in part of things.
The propositions in which we are interested are about things.

For 2,500 years logicians and philosophers have been trying to come to some agreement about what we do or should mean by "thing". We use a vague notion to develop predicate logic. Then by analyzing and formalizing many examples from English and other thing-languages we come to a better understanding of the idea of an individual thing. Similarly, we hope to clarify the conception of the world as process, the flow of all, as we develop the language and logic here. But we can't expect to go as far as we have with the notion of thing, especially since I can't give examples of formalizing from an ordinary mass-process language since I speak none.

In assuming "Things, the World, and Propositions" for predicate logic we do not say that the world is made up only of things. We recognize that there are masses, such as mud and snow. We do not consider it a defect that we cannot formalize much less analyze in predicate logic "Snow is white". That claim is simply outside the scope of that work.

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[7] See my series of book *Essays on Logic as the Art of Reasoning Well* for studies of logic where reasoning well can involve both valid and strong inferences.

But in the view of the world as process, there is no division of the flow of all into things. It is a consequence of that view, not a limitation we adopt to simplify our work, that we will not be able to formalize and reason about things. To see this as counting against the view of the world as process and our work can only be on the assumption that there are things in the world and relations among them that are real.

In all our work setting out formal guides for how to reason well we have incorporated some formalization of the English sentence connectives "and", "or", "not", "if . . . then . . .". Those are ubiquitous in our reasoning in English and in many other languages. Our simplest formalization of them is with classical propositional logic using the symbolizations $\wedge$, $\vee$, $\neg$, $\rightarrow$, as presented in Volume 0. For that logic we assume only that each proposition is (agreed to be) true or false and that the truth-value of the whole is a regular compound of the truth-values of its parts. That abstraction is compatible with our reasoning about the world as process-mass, for it does not matter whether the propositions are about things, masses, process, dogs, mud, or flame. So we can use classical propositional logic in our talk and reasoning about the world as process-mass.

Aside: Possibilities

Some people talk of a way the world could be as if that were a thing, a "possibility". But if possibilities are things, are they physical or abstract? Look around the room where you're reading now. That's a possible way the world could be. It sure seems physical: you can touch the table, see the wall and a light fixture. What about the possibility of the table being moved 50 cm south? That's abstract: it's not part of the physical world. So you move the table, and now it's a physical possibility. So a possibility can change from being abstract to physical. If so, how can we quantify over them? Are there both real dogs and possible dogs?

We "pick out" possibilities with descriptions. We describe as much as we need, and that's what we pay attention to. We have no other choice, for we cannot describe "all of the world", not even all of "the way the world is" in the room where you're reading this.

But what counts as a good description? We say that the description has to be consistent. There is no way this table could be both rectangular and round. But then we're in a vicious circle: what counts as a consistent description depends on evaluating consequences through validity, but what counts as a valid inference depends on what counts as a possibility.

Three ways have been proposed to get out of this circle. Possibilities are real—some abstract and some physical—and we "pick out" those with descriptions. The description is good if it describes a "real" possibility. But how we can tell if it does invariably ends up invoking consistency.

We can say that consistency does not matter. There is a way the world could be in which the table is both round and rectangular. That's what Alexius Meinong suggested, but it's no help in evaluating reasoning.

The other way to get out of the circle is through formal logic. A formal logic gives a clear definition of consistency. Then formal semantics for a formal logic give structural conditions and restrict to a very limited conception of how the world can be, which serves for

what counts as a good description of the world. See my " Possibilities and Valid Inferences " for a fuller discussion.

Aside: *The universality of propositional logic*
I said in *Propositional Logics*:

> It seems hard for me to conceive of someone who reasons who doesn't have a smallest unit of language to which a Yes-No, Accept-Reject dichotomy is applied, and hence to conceive of someone who uses connectives that would not be expressible by the general tables [for $\neg$, $\rightarrow$, $\wedge$, $\vee$]. So perhaps the story of propositional logics I give is universal. For it to be useful for us, for us to develop it, we will probably have to believe that. p. 408

Yet in "The Myth of Language Universals", Nicholas Evans and Stephen C. Levinson say:

> . . . languages differ enormously in the concepts that they provide ready-coded in grammar and lexicon. Languages may lack words or constructions corresponding to the logical connectives "if" (Guugu Yimithirr) or "or" (Tzeltal), or "blue" or "green" or "hand" or "leg" (Yélî Dnye). p. 435

And in "Codifications of Reality: Lineal and Non-Lineal" Dorothy Lee says:

> But the Trobrianders do not describe their activity lineally; they do no dynamic relating of acts; they do not even use so innocuous a connective as *and.* p. xxx

And Lee says in "Conceptual Implications of an Indian Language" that the Wintu do not have "not" as a propositional negator:

> To express negative obligation the *-les* is affixed to the negative auxiliary *ele*(*u*) which means *to not* (*do*), *to not* (*be*).
>
> I translate *eleu* as *to not-do* or *to not-be*; actually, it means: *to not.*
> It is not a negative statement, but rather, a positive assertion of negation. p. xxx

Compare our use of "refrain from" as discussed in my "Reasoning with Prescriptive Claims".

And Jürgen Broschart in "Why Tongan Does It Differently: Categorial Distinctions in a Language without Nouns and Verbs", says:

> In Tongan, '*ikai* '(be) not' is a predicate of its own ('it is not that . . .', see example [*]) and does not help to differentiate the entities in question.
>
> * *'oku 'ikai ke pule'anga*
> PRES NOT SUBJUNCT government
> 'It does not belong to the government.' (lit. 'It is not that it government-s.') p. xxx

Compare how we differentiate between propositional negation and predicate negation in predicate logic (see *The Internal Structure of Predicates and Names*). There are two readings of "Birta is not a cat". The first is to take "not" to attach to the predicate: "Birta is not- (a cat)". For this to be true, Birta must exist: she has some "property", namely not being a cat. The other reading is to take "not" to apply to a proposition: "not: Birta is a cat". This could be true if Birta does not exist. The latter is often rendered in logic texts as "It is not the case that . . .".

The question is whether people using these languages speak and act in any way that *we* would recognize as reasoning.

5 Categorematic Words

Every language has base words, roots, or stems that carry the concepts the speakers of the language use in communicating. They are the *categorematic* parts of speech. They, or rather we using them establish the categories of the language culture. They can stand alone as words, as in Chinese. Or they can be placed in speech only with modifications, with prefixes, suffixes, infixes, as in Salishan languages.[8]

In our work here, I will assume that we have such categorematic or concept words. Some will be grammatically indivisible, the *base* words, that can stand alone to make an assertion, as when pointing to my old dog Birta I say "Dog". Or they can be used in combinations as determined by our grammar. In order to have some for examples, I'll coin versions of from English. For example, we'll have:

DOG-ing	MUD-ing	SNOW-ing
BARK-ing	TALK-ing	CHIMERA-ing
RUN-ing	UNICORN-ing	HATE-ing
CAT-ing	RAIN-ing	LOVE-ing
MEOW-ing	BANANA-ing	FLOWER-ing
SUN-ing	BEAN-ing	ELEPHANT-ing
AUTOMOBILE-ing	JUSTICE-ing	HEROIN-ing

I use all capital letters to remind us that these are not meant to be of any grammatical part of speech of English. And I add "-ing" to relate them to English gerunds that are half noun-half verb, half mass term-half process word, suggesting the movement of the flow of all, though there is nothing to move, nor even to flow, but just all.[9] I sometimes call these *mass-process words*, eliciting how they will be used much as mass and process words in English, or rather as both a mass and a process word. They are waiting for a grammar to show how they will be used, not as nouns or verbs or adjectives but as evoking concepts for describing, but not partitioning, describing without picking out some "part" of the world.

In this list we have words that we as English speakers would consider to stand for masses: "MUD-ing", "SNOW-ing". We have words that we as English speakers would consider to stand for processes: "RUN-ing", "HATE-ing". We have words that we as English speakers would consider to pick out things: "BANANA-ing", "UNICORN-ing". And we have words that we as English speakers have no clear conception of as process, mass, or thing, such as "JUSTICE-ing". It is not that each of these words is a word for a mass or process or even a thing conceived of as a mass or process. We are to think of them as describing the flow of all, process, in a mass-

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[8] See my essay "Nouns and Verbs" in *Language and the World* for a fuller discussion.

[9] If we were to borrow from Italian we'd use infinitives, which are used like gerunds in English.

like way. "BANANA-ing" is a concept word of banana as mass or process as much as "MUD-ing"; "JUSTICE-ing" is a concept word of justice as mass or process as much as "RUN-ing". From our thing-language perspective, "DOG-ing" can describe dog-ing, essence of dog, a dog, dogs, and more. In that respect, these are closer to their Latin equivalents.

Categorematic words *Categorematic words* evoke the concepts or categories of the language. A *base* categorematic word, or simply a *base word* is one that is grammatically indivisible.

Using a base categorematic word, we hope to describe some of the flow of all, to direct someone's attention. When I say "RAIN-ing" and point out the window, that utterance is true or false. That's because the context indicates that I'm talking about now and outside the window. By itself, "RAIN-ing" is just a way to describe, not a description. Some context, either physical or with other words, is needed to make a base word into a proposition.

In saying "RAIN-ing" while pointing out the window, when that's true I do not direct someone's attention to a part of the flow of all. That would be to take parts of the flow of all as things, yet raining is not a part of the flow of all. There is only my pointing to establish a context and I have used the way of describing the flow of all with "RAIN-ing" correctly: it is true.

So a base word is not by itself true or false; it is true or false given a context. A context can be established in many ways. For example:

"RAIN"	pointing	is true or false
"DOG"	touching a dog	is true or false
"SMOKE"	smelling	is true or false
"WIND"	feeling the breeze	is true or false
"SHEEP"	saying "At the corral last week"	is true or false

It's tempting to say that a context is a possibility. But that seems odd to say about these examples. Rather, establishing a context is like picking out a possibility. But that, too, seems odd, for what is the possibility when smelling? The notion of a categorematic word being true in, or of, or relative to a context seems much more physical and behavioral than any idea we have of possibility. I hope to have led you to understand that enough to take it as a primitive semantic notion that we will explore in this volume.

Establishing a context, or simply a context, is not meant to be understood as a thing. When I get you to smell, that's not a thing. Yet in what follows, I'll talk of all contexts in which, say, "ELEPHANT-ing" could be true, which seems to depend on

thinking of contexts as things. I'll talk of some context in which "HATE-ing" could be true, which suggests I'm talking of contexts as things. But that needn't be, any more than when I talk of all mud I am intending for you to think of "muds" as things, or when I talk of some running I am intending for you to think of "runnings" as things.

So in what follows I will talk of a context, of some context, or of all contexts in which a categorematic word could be true. And speaking English to you, I'll usually describe a context using thing-talk.

Names

If all is process, and talk is talk of the flow of all, then names in English, the quintessential words for things, are ways of describing in the flow of all. What is Zoe? She is a woman; but that's to talk of her as a thing. She doesn't like to be talked of as a thing. She sees herself as more than that, as something that continues in time, has continuity, is a process. Being true to that vision, we treat her name as a mass-process word.

Thinking of the world as made up of things, we have to figure out what unifies all the life of Zoe. What is it that makes Zoe last year the same as Zoe this year, though not the same but the same "thing"? Do we talk of an "instance" of Zoe? A "time-slice" of Zoe? So we have the time-slice of Zoe at August 2, 2003. But that's a pretty big time. There's the time-slice of Zoe at 10:22 a.m. August 2, 2003. And "smaller and smaller" time-slices. And that looks very much like taking parts of a mass. Viewing the world as process-mass, "ZOE-ing" is a word that we can use to distinguish in the flow of all; there is no question of unifying the "parts" of Zoe. Zoe-ing is a mass that is unified no differently than water. We can talk of some Zoe-ing, and that way of talking fits better into the idea of time-slices than the thing-talk. The idea that Zoe is a thing is quite bizarre, absurd, an idea whose justification is only to use nouns and verbs well in English (and other thing-languages). There is a whole there, but not one made up of parts, a unity that we know as much as we know the unity of the water in this lake is not the same as the water in this other lake.

In what follows, then, I will use as base categorematic words English words that we as thing-talkers would consider to be names.[10]

Aside: Concepts

When I say that "dog" establishes a concept for us to communicate, I am not thinking of "a" concept as some fixed thing the word stands for or represents. I intend it as Benson Mates describes in *The Skeptic Way*: *Sextus Empiricus'* Outlines of Pyrrhonism:

> Suppose now that someone were to ask the Socratic-style question, "What *is* a concept?" A standard answer in Sextus's day would be that a concept is a change

[10] In "On What There Is" W. V. O. Quine seemed to come to this view of proper names. He said that in classical predicate logic we could replace proper names by predicates. So we could use "pegasizes" rather than "Pegasus". This avoids problems with non-referring proper names, but does not seem consonant with the foundations of predicate logic in which the roles of names and predicates are meant to be quite distinct. See the discussion in Chapter VIII of *Predicate Logic.*

> of state (*kinesis*) of the intellect (M 8.336a).[11] But a somewhat more informative answer, which can be inferred from the use of the terms in actual practice by Sextus and his opponents, is this: a concept is in effect the meaning of a word or noun phrase. To have a concept of human being (*anthropos*) is to know what a human being is (M 8.87ff.); that is, it is to grasp a sense or meaning that is expressed in Greek by the term *anthropos*. Thus when Sextus raises the question of whether *X* is conceivable, he is in effect questioning the very meaningfulness of utterances containing the corresponding word, insofar as that word is to be used in accord with one or another of the Dogmatists' definitions. . . .
>
> However, if we allow that for Sextus and his opponents concepts are in effect the meaning of words, it must be added that these meanings are not to be treated in the Fregean manner as independently existing ideal entities that may be expressed with varying degrees of accuracy by the words we use. Instead, they are to be thought of more "psychologically." Concepts are said to be "formed" by people on the basis of their individual experiences; consequently, since different people will mean different things by a given word, we can expect to be told that they will have different concepts associated with that word. pp. 22-23

This is how I talk of meaning in "Language-Thought-Meaning".

Aside: Parts of the flow of all?

Suppose that we were to think of parts of the flow of all as what makes a mass-process word like "RAIN-ing" true or false. That would be to divide the flow of all into things. And that won't do. It's not just that smelling to establish a context does not pick out part of the flow of all. It's that we have no way to conceive of the flow of all divided into parts because we cannot identify particular parts of the flow of all.

The problem of taking parts of the flow of all to be things is like the problem of taking parts of space as things, as discussed in Volume 2. Space is a mass. Yet to establish a formal logic of things in time and space in Volume 2 we had to adopt the pretense that we could describe and pick out parts of the mass of space to treat as things. That is more comfortable for us because of our habit of marking off parts of space, either physically with stones, or noting the location of a patch of cactus, or by using coordinates.

Can't we take parts of the flow of all to be (like) events? Those are things. No, events are not things, as I've explained in "Why Event-Talk Is a Problem". The only way we can pick out an event is with a description. "The stabbing of Julius Caesar" supposedly describes an event, but whether it's the same event as "The stabbing of Julius Caesar with a knife quickly" is a question we try to ponder with no clear answer. In contrast, we have clear (enough) criteria for whether "Birta the dog of Richard L. Epstein" and "the dog that is in the patio of Richard L. Epstein's ranch on June 6, 2009" describe the same thing. When we do not have clear criteria, we doubt that we are talking about what can be conceived of as things.

But can't we assume that we have (or there exist) complete descriptions of events and similarly of parts of the flow of all? In Chapters 35 and 37 of Volume 2 we saw that we cannot specify a minimal location for a particular dog, a place where there is just the dog. And if things don't determine locations, what more specific could we use? There is no

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[11] [M = *Against the Mathematicians*]

unique place where “Birta is running on July 10, 2012” is true, even if Birta ran only once that day. There is no description that is complete enough to pick out in a context just that part of the flow of all that is run-ing other than to point and say “RUN-ing”.

In what follows, I will not speak of a part of the flow of all. Contexts, yes; some of the flow of all, yes; parts of the flow of all, no.

6 Conjunctions of Categorematic Words

I point and mean to get you to notice that my dog Birta is running. I might point and say "DOG-ing and RUN-ing" and mean that the two are joined. It's not just that in the direction I'm pointing "DOG-ing" can describe correctly and "RUN-ing" can describe correctly, for that could be (in our thing-talk) if there were a dog sleeping and a woman running. No, I mean for the two descriptions to describe together.

It is tempting to say that the process of dog-ing and the process of run-ing were intertwined, meshed to be one process. But to talk of an intertwining or meshing is to treat "DOG-ing" and "RUN-ing" as names of processes, and hence as things. They are not names: there is only the flux of all, and these words used to describe. Together "DOG-ing" and "RUN-ing" can be used to describe the flux of all in that context. That "together" is what distinguishes dog-running from burro-running and dog-sleeping. There is a difference between "DOG-ing" and "RUN-ing" applied separately or together as a description of a time and place, and the difference in the description comes from a difference in the world. But we find it hard to say more than that, and we need not. We do not need to give a metaphysics of processes combining, and trying to do so leads me into thing-talk. We need only recognize a part of the flux of all as allowing for a description combining categorematic words. And we need a way to write that.

Let's use the symbol "+ " when we want to conjoin two categorematic words as a description. So we can have:

CAT-ing + MEOW-ing

JUSTICE-ing + BARK-ing

ELEPHANT-ing + MUD-ing

RUN-ing + DOG-ing

The last could be used for what we would call in English a dog running, or some running done by a dog, or a pack of dogs running, or the concept of dog-running. Pointing to my dogs chasing a rabbit, this would be true. Pointing to my patio now, though Birta is there, it would be false. Pointing to a marathon race with no dogs in sight, it would be false. Even pointing to a marathon race with a dog looking on, it would be false: the dog-ing and the running are not mixed.

The suffix "-ing" is useful for reminding us of our new way of using words, but it is distracting. I'll use it now only when I want to emphasize the difference of our language from English. You'll remember, I hope, that "RUN" is not meant to be read as a verb, and "DOG" is not meant to be read as a noun (or vice-versa).

So consider:

DOG + HUMAN

Surely this can never be correctly asserted about any part of the flow of all. It is vacuous. After all, we can't mate. But that's to think in thing-talk. Yes, no object can be both dog and human. But "DOG + HUMAN" correctly describes me when my dog Birta is walking alongside me in the hills, or when she and I are playing, rolling around on the ground, or when I am petting her stomach. It is easy to slip into thinking of the world as made up of things.

There is a temptation to think of a conjunction of categorematic words as a kind of predication. "DOG + RUN" is like asserting "A dog is running". But equally it would be true in a context of many dogs running. And equally it serves as a way of paying attention to the flow of all, as a concept word. And certainly "SNOW + WHITE" is not a predication in the sense used in thing-talk.

Here is an example that we can't construe in thing-talk:

SALT + WATER

This can be true about the mixture I get if I start with water and add some salt or start with salt and add some water. All that matters is that the part of the flow of all can be described by both "SALT" and "WATER" together. The order of the terms doesn't matter any more than it does in "DOG + HUMAN". If a conjunction of categorematic words is a correct description, then the categorematic word with the parts reversed is a correct description, too: they establish the same concept.

We can conjoin more than two categorematic words:

(CAT + LOVE) + HUMAN

CAT + (LOVE + HUMAN)

These are not equivalent descriptions. The first could be true (describing in thing-talk) if a cat is loving a woman who has no interest in the cat but is simply holding it in her arms for a friend who's gone to answer the telephone. But for the second to be true, the human-ing must be mixed with the loving, while the cat could be quite indifferent, which is the normal state of affairs. And neither of these is equivalent to:

(1) CAT + LOVE + HUMAN

Here cat-ing is mixed with loving and human-ing, which would describe my friend who is petting her loving cat that is sitting on her lap. We need to allow for conjunctions of more than two categorematic words both with and without internal parentheses. We can then join a conjunction without parentheses with a categorematic word:

(CAT + LOVE + HUMAN) + DOG

This would be true of the same context as described for (1) if there is also a dog sitting next to the woman trying to get the woman to pay attention to him.

If (1) is true, then so is "CAT + HUMAN + LOVE". As I prepare my soup, the mixture is described equally by "SALT + WATER + VINEGAR", "SALT + VINEGAR

+ WATER", "WATER + SALT + VINEGAR". Though the salt and vinegar cannot be extracted from the mixture as a separate combination, they are both there in a mixture. In all conjunctions the order does not matter.

To make that precise and give a definition of a conjunction of categorematic words, we need some symbols to stand for categorematic words. Let's use:

E, F, E´, F´, E_1, E_2, . . .

But don't the numerals introduce a notion of thing? We use numerals to count, and it's things we count. The numerals are only for our convenience as thing-language speakers. We could accomplish indexing by using colors. Imagine:

E(black), E(light grey), E(very light grey), E(royal blue),
E(navy blue), E(sky blue), E(chartreuse), E(puce), E(rose), . . .

Doing so would not introduce a notion of thing.

Conjunctions of categorematic words If $E_1, \ldots, E_n$ are categorematic words, then $(E_1 + \ldots + E_n)$ is a categorematic word. It is a *conjunction of categorematic words*; each E_i is a *conjunct*.

The conjunction $(E_1 + \ldots + E_n)$ is a good/accurate/correct description if each conjunct is a good/accurate/correct descriptions and all of the conjuncts are used to describe in some mixed, or joined, or together way.

We read the symbol "+" as "plus".

What we mean by "some mixed, or joined, or together way" will depend on the particular words that are being conjoined. We understand the together of "DOG + RUN" differently from the together of "SHOE + FOOT". Yet there seems to me to be some underlying, general notion of two categorematic describing together.

I'll let you give more examples of a conjunction of categorematic words used in a context in which it is true and used in a context in which it is false. But I suspect that you, as I, will be able to set out those contexts only using thing-talk.

Aside: Chinese without conjunctions

In "Nouns and Verbs" we saw how Chinese characters can be used in juxtaposition to say quite a lot without any connectives, indications of time or location, or words to indicate a comparison. Those characters are like our categorematic words. In the passages there they were used in poetry, the lines of which are not meant as propositions. Still, those passages suggests that we could use categorematic words in juxtaposition to make propositions. Jaques Gernet in *China and the Christian Impact: A Conflict of Cultures*, Jaques Gernet describes what principles Chinese speakers use to combine words.

> Given that Chinese is an uninflected langauge, all that helps to guide one through a phrase, with the aid of a very limited number of particles, are the links between terms of similar meaning, the oppositions between terms of opposite meaning, the rhythms and

> parallelisms, the position of "words" or semantic units and the types of relationship between them; and yet the infinite possible combinations of two semantic units are the source of an inexhaustible fund of meaning. At every level, meaning stems from the way terms are combined. No doubt this is what accounts for the predominant role played by complementary pairs of opposites and correspondences in Chinese thought and above all for its fundamental relativism. Nothing has meaning except through opposition to its contrary. Everything depends upon position (*wei*) and timing (*shi*). p. 242

See also "Conceptual Schemes and Linguistic Relativism in Relation to Chinese" by A.C. Graham, in *Culture and Modernity: East-West Philosophic Perspectives*, ed. Eliot Deutsch, University of Hawaii Press, 1991, pp. 193–212, for an explanation of how the system of similar and opposite meanings works in Chinese.

The passages in "Nouns and Verbs" in word-for-word translation are difficult for us because we don't know the allusions or the way of life the poems are meant to reflect. At the risk of being ridiculous, I offer here four poems in that style to give an idea how using categorematic words by themselves can convey a great deal and in the hope, probably quite forlorn, that these will help us appreciate Chinese poetry more.

DOG BARK MIDDLE NIGHT
COCK VILLAGE DAWN
SLEEP ABSENT SLEEP ABSENT
WOOF CREE-CREE

LEAF WATER FLOAT
FISH GULP
OOPS

SUN HOT SHINE HOT SWEAT
TREE BRANCH SPIKE SHADE-NO
SHE VOICE WATER ROCK RIPPLE
DRINK DEEP

DOG BARK
CAT RUN
LIFE GOOD

7 Adjectives and Adverbs?

All the base categorematic words we've been using have been derived from English nouns and verbs. Yet we'd like a way to assert that there outside the window is white snow-ing, or there in my friend's house is gentle cat-ing. Can we find a way to use adjectives and adverbs from English in our mass-process language?

The phrase "white snow" tells us what kind of snow. But what difference is there between saying that there (pointing) is snow-ing and there (pointing) is whiteness? We can say "white-ing" to describe some of the flow of all, viewing white-ing as process as much as dog-ing. Whiteness is in the world, as much a mass (concept) as mud, as much a process (concept) as running. We can treat color words as base categorematic words: they divide up, describe, or make a distinction in the flow of all. In this context there is white-ing. In that context there is red-ing. All is flow in the flow of all. We can adopt the following as base categorematic words:

WHITE	GREEN
RED	YELLOW
BROWN	CHARTREUSE

So to say that there is white snow, we can use "WHITE + SNOW" or equivalently "SNOW + WHITE". There (pointing) snow-ing and white-ing serve as descriptions mixed together, and one word does not have precedence over the other.[12]

I can point and assert that there is red hat-ing or hat red-ing with "RED + HAT" or equivalently "HAT + RED". I can point and assert that there is yellow sun-ing with "YELLOW + SUN". We can see yellow-ing in the world, as if the sun is not yellow but is a process, sun-ing, mixed with yellow-ing. It might seem that in these examples the color words are conjoined with only base categorematic words derived from English nouns. But "SUN" is not a noun, nor is it a verb. It describes in the flow of all as much as "RUN". We can use "COLOR", too, as a base categorematic word. It's more general, yet meaningful in the same way.

What about "gentle"? As English speakers, we could understand six dogs or a dog running or a pack of dogs as gentle. Would that be a comparison to only other dog-ing? Surely "gentle" means the same for a gentle dog and a gentle elephant. But does it mean the same for a gentle boa constrictor? For a gentle ferris-wheel ride? To say no is, I suspect, to think of "gentle" as needing a basis for comparison —of things. But even in English we have a mass-word for the idea of being gentle:

[12] Compare W.V.O. Quine, *Word and Object*:

> In attributive position next to a mass term the adjective must be taken as a mass term: thus "red" in "red wine". The two mass terms unite to form a compound mass term. When we think of the two component mass terms as singular terms naming two scattered portions of the world, the compound becomes a singular term naming that smaller scattered portion of the world which is just the common part of the two. p. 104

"gentleness". A big problem in treating "gentle" as a modifier of predicates in Volume 1 is how to discern a general notion of gentle from its disparate uses modifying "— is a dog", "— is an elephant", "— is a boa constrictor", "— is a ferris wheel". Here we can say that "gentle" gives a concept that we can use in combination with other categorematic words; a comparison is not needed. We can take "GENTLE" as a base categorematic word. Then we can say "GENTLE + CAT" or equivalently "CAT + GENTLE": there (pointing) is cat-ing plus gentle-ing, or equivalently gentle-ing plus cat-ing.

Similarly, in English we have mass forms of "loud" ("loudness") and "strong" ("strength"). We can take "LOUD" and "STRONG" as base categorematic words, too, and have:

BARK + LOUD

JUSTICE + STRONG

We also have "beauty" as the mass form of "beautiful". So we can take "BEAUTY" as a base categorematic word. Yet isn't that platonizing? Isn't that to take beauty as a universal, independent of the kind? Yes, in thing-talk it would be. But here we are not comparing things of a kind, saying that "beautiful" is meant differently in the comparisons "beautiful woman" and "beautiful dog". There is only "BEAUTY" that can be a good mass-process description of some of the flow of all. We have the concept, but not necessarily an abstract or platonic universal.

What about the adjective "big"? What would it mean to say that "BIG" divides up the flow of all? Is there "big-ing" here and now? We don't have in English a mass form of "big", not "bigness" for sure. If I say "Bidú is big", you'll want to know what kind of thing Bidú is. He's a dog, and yes, he's big compared to other dogs. He's an animal, and no, he's not big compared to other animals like bulls, elk, and elephants. What is big depends on the kind of things we're talking about. In the view of the world as process, there are no things and no kinds of things. "DOG" can correctly describe some part of the flow of all that we as speakers of English would say is two dogs, or six dogs, or a pack of dogs, or two dogs running, or . . . . What would it mean to say in such cases that "big" is a correct description? Adjectives we use in English to compare a thing to other things of a particular kind are not suitable to use as categorematic words.

Words like "very", "nearly", and "almost" whose role in English is to modify other adjectives and adverbs also don't make sense to use as categorematic words. We'll consider them at the end of the Chapter 10.

In general, then, we can adopt adverbs and adjectives from English to use as base categorematic words as long as their use and our understanding of them is not solely for comparison, as with "big", or used solely (primarily?) as modifiers of adjectives or adverbs, as with "very".

Aside: Colors as process

Both Norwood Russell Hanson and Friedrich Waismann, apparently without being aware of each other's work or that of Benjamin Lee Whorf and other linguists, also suggested that we could colors as process.

In *Patterns of Discovery* Hanson says:

> That it is yellow is a passive thing to say about the sun, as if its colour were yellow as its shape is round and its distance great. Yellow inheres in the sun, as in a buttercup. 'The sun yellows', however, describes what the sun does. As its surface burns, so it yellows. Now the grass would green; it would send forth, radiate greenness—like X-ray fluorescence. Crossing a lawn would be wading through a pool of green light. Colleges would no longer be cold, lifeless stone; now they would emit greyness, disperse it into the courts. As a matter of optics this is rather like what does happen; the change of idiom is not utterly fanciful.
>
> . . . Speaking of colour-words as verbs just is to think of colours as activities and of things as colouring agents.
>
> What if information about colours were expressed adverbially? We would then say 'The sun glows yellowly', 'The grass glitters greenly', 'The chapel twinkles greyly'. If everyone spoke thus how could one insist on its being a fact that the sun is yellow, that grass is green, or that the chapel is grey? Could such 'facts' be articulated at all? . . .
>
> What of primary qualities? 'The sun is round' states a fact. So too 'St John's College hall is rectangular', 'sugar lumps are cubes'.
>
> Try 'the sun rounds', 'St John's hall rectangulates', 'sugar cubes'. Activity is suggested here. Would one who saw the round sun see the sun rounding? The college hall *is* rectangular. Would this fact be apprehended by a man for whom the hall rectangulates—holding itself in a rectangular form against gravity, wind, cold and damp? Perhaps the man for whom the sun rounds would see the sun incessantly arranging itself as a sphere. If he can say only 'The sun rounds', how else can he see it? pp. 33–34

In "Verifiability" Waismann says:

> People are inclined to think that there is a world of facts as opposed to a world of words which describe these facts. I am not too happy about that. Consider an example. We are accustomed to see colours as a "quality" of objects. That is, colour cannot subsist by itself, but must inhere in a thing. This conception springs from the way we express ourselves. When colour is rendered by an adjective, colour is conceived as an attribute of things, i.e., as something that can have no independent existence. That, however, is not the only way of conceiving of colour. There are languages such as Russian, German, Italian, which render colour by verbs. If we were to imitate this usage in English by allowing some such form as "The sky blues", we should come face to face with the question, Do I mean the same fact when I say "The sky blues" as when I say "The sky is blue"? I don't think so. We say "The sun shines", "Jewels glitter", "The river shimmers", "Windows gleam", "Stars twinkle", etc.; that is, in the case of phenomena of lustre we make use of a verbal mode of expression. Now in rendering colour phenomena by verbs we assimilate them more closely to the phenomena of lustre; and in doing so we alter not only our manner of speaking but our entire way of apprehending colour. We *see* the blue differently now—a hint that language affects our whole mode of apprehension. In the word "blueing" we are clearly aware of an active, verbal element. On that account,

"being blue" is not quite equivalent to "blueing", since it lacks what is peculiar to the verbal mode of expression. The sky which "blues" is seen as something that continually brings forth blueness—it radiates blueness, so to speak; blue does not inhere in it as a mere quality, rather it is felt as the vital pulse of the sky; there is a faint suggestion of the operating of some force behind the phenomenon. It's hard to get the feel of it in English; perhaps it may help you to liken this mode of expression to the impressionist way of painting which is at bottom a new way of seeing: the impressionist sees in colour an immediate manifestation of reality, a free agent no longer bound up with things.

There are, then, different linguistic means of rendering colour. When this is done by means of adjectives, colour is conceived as an attribute of things. The learning of such a language involves for everyone who speaks it his being habituated to see colour as a "quality" of objects. This conception becomes thus incorporated into his picture of the world. The verbal mode of expression detaches colour from things: it enables us to see colour as a phenomenon with a life of its own. Adjective and verb thus represent two different worlds of thought.

There is also an adverbial way of rendering colour. Imagine a language with a wealth of expressions for all shades of lustre, but without adjectives for colours; colours, as a rule, are ignored; *when* they are expressed, this is done by adding an adverb to the world that specifies the sort of lustre. Thus the people who use this sort of language would say, "The sea is glittering golden in the sunshine", "The evening clouds glow redly", "There in the depth a shadow greenly gleams". In such phrases colour would lose the last trace of independence and be reduced to a mere modification of lustre. Just as we in our language cannot say "That's very", but only some such thing as "That's very brilliant", so in the language considered we could not say "that's bluish", but only, e.g., "That's shining bluishly". There can be little doubt that, owing to this circumstance, the users of such language would find it very hard to see colour as a quality of things. For them it would not be the *things* that are coloured, rather colour would reside in the lustre as it glows and darkens and changes—evidence that they would see the world with different eyes.

"But isn't it still true to say that I have the same experience whenever I look up at the sky?" You would be less happy if you were asked, "Do you have the same experience when you look at a picture puzzle and see a figure in it as before, when you didn't see it?" You may perhaps say you see the same lines, though each time in a different arrangement. Now what exactly corresponds to this different arrangement in the case when I look up at the sky? One might say: we are aware of the blue, but this awareness is itself tinged and coloured by the whole linguistic background which brings into prominence or weakens and hides certain analogies. In this sense language does affect the whole manner in which we become aware of a fact: the fact articulates itself differently, so to speak. In urging that you *must* have the same experience whenever you look at the sky you forget that the term "experience" is itself ambiguous: whether it is taken to, e.g., include or to exclude all the various analogies which a certain mode of expression calls up.[13] pp. 54–56

[13] As suggestive as Waismann's examples are, he is wrong about the use of "blue" as a verb in German and Russian. Neither in German nor Russian is there any sentence we could understand as "The sky blues" in a continual sense of being blue. Russian does allow color words to be turned into verbs as incipients, as we do in English with "to redden" or "to green". We can say "Her face is reddening" for "Her face is turning (becoming) red"; we can say "The field will green" for "The field will turn green". But those uses are not what Waismann is suggesting. However, speakers of Koyukon do speak of color much as Waismann describes: see Melissa Axelrod, *The Semantics of Time*: *Aspectual Categorization in Koyukon Athabascan*, p. 129.

8 Subordination

In English we talk of some mud, this mud, that mud, parts of mud. Basic to our talk of masses is the idea of a part of a mass. But the word "part" is wrong. It's not just that it suggests physical parts. It's that it's used to contrast with a whole, and a whole is a thing. With masses there is part but no whole. The mud in my patio is part of all mud, and all mud is not a whole. The word "MUD" does not pick out a whole.

The word "ANIMAL" also does not pick out a whole. It can be used to describe in many contexts of the flow of all; it is mass-process as much as what we describe with "MUD". Animal-ing here and now is "part of" what we understand with "ANIMAL". In particular, the dog-ing in my patio is part of animal-ing. In any context in which I can correctly assert "DOG", it is also correct to assert "ANIMAL". We can view that as a deep metaphysical observation about the nature of dog-ing and animal-ing. Or we can see it as an observation about how to use those words correctly. Or we can see it as a relation between the concepts of dog-ing and animal-ing. It is not a part-whole relation nor an inclusion relation, for in the mass-process view of the world, there are no wholes and no mental or platonic entities that could be included one in another. Or at least none are needed. I would rather call it *subordination* of ways of paying attention to the flow of all.

Subordination For categorematic words E and F, we write "E sub F" for "E is *subordinate* to F". It is true if and only if the way of paying attention to the flow of all given by E is included in the way of paying attention to the flow of all by F. It codifies a correct use of words.

So the following are propositions:

WOMAN sub MAMMAL

BANANA sub FRUIT

SNOW sub WHITE

WHITE sub COLOR

FLOWER sub LOVE

JUSTICE sub VIRTUE

Each of these is true or false. They are not about the words, relating "WOMAN" to "MAMMAL", for then we should write: "WOMAN" sub "MAMMAL". They are about ways of paying attention to the flow of all, which are concepts, ways of seeing, so long as you do not think of concepts as things, especially not disembodied things.

The proposition "BANANA sub FRUIT" is true: banana-ing is "part of", "included in", subordinate to fruit-ing. As a consequence, in whatever context

it is correct to assert "BANANA", it is also correct to assert "FRUIT". In contrast, "FLOWER sub LOVE" is false: flower-ing is not "part of", is not "included in", is not subordinate to fruit-ing. I can point to a daffodil in a field without any animals and say "FLOWER", and that's correct, but "LOVE" is not correct to assert.

It is tempting to understand subordination in terms of when it is correct to assert categorematic words:

(1) E is subordinate to F iff
in any context in which it is correct to assert E, it is also correct to assert F.

That would reduce subordination to the semantic primitive we've already adopted: truth in a context. But then consider:

PEGASUS sub DOG

By (1), this would be true because there is no context (in the world as we know it) in which it is correct to assert "PEGASUS". Yet in terms of paying attention to the flow of all, the concept elicited by "PEGASUS" is not part of the concept elicited by "DOG". We have, rather, "PEGASUS sub HORSE".

We are at a divide that medieval logicians and metaphysicians faced. When we assert "Homo est animalum", are we talking about concepts or are we talking only about what can be referred to or picked out by the words "homo" and "animalum"? The nominalists rejected universals and said that "Homo est animalum" is true iff every thing that can be correctly described by "homo" can also be correctly described by "animalum". That's like the equivalence at (1). But even they, even the arch-nominalist Buridan, had recourse to mental concepts to justify why that should be. They avoided classifying "Pegasus est canis" as true by excising the default option: for "E est F" to be true, there must be some thing, some individual of which E is true. We could go that route here, taking (1) as the condition for subordination. But to do so would be to eliminate the following as more than vacuously valid:

PEGASUS sub HORSE
HORSE sub ANIMAL
Therefore, PEGASUS sub ANIMAL

It would be to eliminate any talk of ideas or concepts as part of our informal semantics. Only a strong commitment to what I would call *mass-process nominalism* would be motive for doing that.[14]

I think that the definition of subordination above, talking of ways of paying attention to the flow of all understood as invoking ideas or concepts, is the better way to go. The truth-in-a-context condition is a consequence of but is not sufficient for there to be subordination.

[14] See the discussion of the ambiguity of using "est" for a genus-species relation and as predication in medieval logic compared to philosophical analyses in the mass-process language Chinese in Appendix 2 of *Disputers of the Tao* by A. C. Graham.

Subordination and correct use of a categorematic word in a context
If "E sub F" is true, then in any context in which E can be used correctly to describe, F can be used correctly to describe.

It may be that the way we come to accept or recognize a subordination relation is through considering in what contexts a categorematic word can be correctly asserted, invoking different ways the world could be according to the metaphysics we adopt. That is how I will often justify a subordination proposition in the discussions that follow. But subordination cannot be reduced to talk of truth in contexts, though it is "expressed" in such talk. Subordination is about concepts, or correct use of words, or ways of paying attention to the flow of all.

But suppose that some baby, Houle, is born and cries from the moment of its birth until it dies two days later.[15] Then in every context in which we could correctly assert "HOULE", we could correctly assert "CRY". Or not, depending on whether "any context" means in the world as it is or as it might be. Perhaps Houle could have been born to a rich Saudi prince and hardly ever cried, so that there are contexts in which we could correctly assert "HOULE" and it would be incorrect to assert "CRY".

The difference between "all" contexts and contexts restricted to some "possibility", say the way the world is and has been, is reflected in what is true in all models and what is true in only some models. But we can't take "DOG sub ANIMAL" as an axiom true in all models for it can't be distinguished in form from "HOULE sub CRY", and it is formal logic we are building. We may take "DOG sub ANIMAL" as true in every model we consider, but if so it is because of what we believe about the ways of dividing up the flow of all that are given by "DOG" and "ANIMAL". Moreover, we want to be able to disagree, for some to say that "DOG sub ANIMAL" is true and others to say it is false and compare those views by looking at different models.

But what of the assumption in all of logic that in any discussion, any reasoning we do, we use words the same: words are types that do not change their meaning. In formal logic, what we mean by "in a discussion" is captured by a model, and there "DOG sub ANIMAL" has only one truth-value. We as logicians, as people concerned with the forms of language and meaning, are not setting ourselves up as arbiters of truth, as saying what is true and what is false in this our world as it is. That would be to take logic back to the days when there were no models, only a semi-formal language meant to codify all truths.

The subordination relation is primitive. We classify "DOG sub ANIMAL" as true and "HOULE sub CRY" as false in our understanding of ways of paying attention to the flow of all, in terms of concepts. But those classifications are only relative to us, now, as we conceive and use those words. Those classifications could be different, and that is what we mean by taking a model.

[15] This could well happen in Yemen where the Saudis, with the guidance and support of the United States, have bombed the infrastructure, including hospitals, and kept food from reaching the people.

Example 1 FISH sub WATER

Analysis Suppose a child asks whether this is true. Yes, the mother says, and by doing so excludes the fish on a dinner plate as part of fish-ing and excludes "FISH" from describing correctly a fish flopping on the beach. In that case, she'll need a word other than "FISH" for what we call a fish out of water. Thus the example, which seems to be an assertion about how the world is or a relation of ways of dividing up the flow of all could be construed as a way to constrain the meaning of "FISH".

Example 2 HORSE sub COLOR

Analysis With our "usual" understanding of these words, this is true. Whenever I can point and correctly describe with "HORSE", I can correctly describe with "COLOR". Conceiving of horse-ing entails conceiving of color-ing. This is clearly not a part-whole relation. In the Western tradition, philosophers argue whether color is an attribute of horses, or whether color is part of the essence of being a horse, or whether color is of a horse. Modern logicians evade these debates by reading the example as "If anything is a horse, then it's colored", reducing all to thing-talk. We can't use the conditional "If HORSE, then COLOR" because "HORSE" and "COLOR" are not propositions. We have only "HORSE sub COLOR".

Example 3 DOG sub (CANINE + DOMESTIC)

(DIG + SHOVEL) sub WORK

Analysis Conjunctions of categorematic words can also be in a subordination relation.

Aside: Concept inclusion

"DOG sub ANIMAL" is true. But some contend this is backwards. What we should have is "ANIMAL sub DOG" is true, for the concept of animal is contained in the concept of dog. Whenever I try to understand this, I just end up with the converse of the subordination relation, which you could start with if you prefer.

In Chinese writing:

the character for *dog* includes the radical for *animal*.

the character for *fox* includes the radical for *animal*.

the character for *lion* includes the radical for *animal*.

Perhaps for Chinese speakers it is more natural to take the converse of subordination as primary.

Aside: Concepts versus contexts

The distinction between subordination as a condition on concepts versus a condition on ways that categorematic words can be used correctly is very similar to what Mates describes in *The Philosophy of Leibniz*:

> The first of these [differences from other commentators on Leibniz] results from giving due attention to Leibniz's distinction between "essential" and "existential" propositions. Any proposition '*A* is *B*', he tells us, is equivalent to the corresponding proposition '*AB* is an entity'. Thus, "Caesar is Roman" is equivalent to "Caesar the Roman is an entity." But the word "entity" is ambiguous; it can mean the same as "possible thing" or it can mean the same as "actually existing thing." If it is taken in the former sense, the propositions concerned are called "essential"; if it is taken in the latter sense, they are "existential."
>
> Clearly the two kinds of propositions have different truth conditions. The essential proposition "Pegasus is a winged horse" is true, since Pegasus the winged horse is a possible thing; but the existential proposition expressed by the same words is false, for Pegasus the winged horse does not actually exist. In general, existential '*A* is *B*' is false if '*A* exists' is false, whereas this is not necessarily the case with essential '*A* is *B*'.
>
> This distinction gives us a way of reconciling some important Leibnizian texts that have seemed inconsistent with one another. Thus, sometimes he says that '*A* is *B*' is true only if the concept *B* is included in the concept *A*, while elsewhere he says that '*A* is *B*' is false if no actual entity falls under *A*. In particular, sometimes he indicates that all propositions of the form '*A* is *A*' are necessary truths, while in other texts he proposes to represent truths of the form 'No *A* is *B*' as '*AB* is not *AB*'. p. 9

Aside: Comparing our mass-process language to English

Consider

(*) JUSTICE sub VIRTUE

Translating this to English perhaps we should use:

Justice is virtue.

But that's not right, for in this "is" has to be read as identity. Perhaps we should say:

Justice is a virtue

But then virtues are viewed as things, and justice is one of them, which can't be right. Nor can we translate (*) as:

Whatever is just is (a) virtuous (thing).

That's almost certainly false, whereas "JUSTICE sub VIRTUE" is almost certainly true.

In English, we're forced to ask:

Is justice a thing?

Does justice exist?

That's because "justice" is a noun. Both questions are nonsense in a mass-process language. Indeed, they can't even be said.

9 Principles of Subordination

Using our informal understanding of the world as process, what subordination principles should we adopt in building a formal logic?

Reflexivity of subordination E sub E

Transitivity of subordination If (E_1 sub E_2) and (E_2 sub E_3), then (E_1 sub E_3).

I can't justify these beyond pointing to the idea of subordination as inclusion of ways of describing the flow of all or as codifying correct uses of words. These principles are fundamental, and more examples won't make them more plausible.

Equivalence
Suppose that "E sub F" and "F sub E" are true. Then whenever we can use E to describe correctly, we can use F to describe correctly; and whenever we can use F to describe correctly, we can use E to describe correctly. Does that mean they are the same? No, for both of the following are true:

DOG sub (CANINE + DOMESTIC)

(CANINE + DOMESTIC) sub DOG

"DOG" and "(CANINE + DOMESTIC)" are not the same: they are equivalent ways to divide the flow of all. We could take this notion of equivalence as fundamental, but we need not, for we can make the following definition.

Equivalence of subordination $E \approx F \equiv_{Def}$ (E sub F) $\wedge$ (F sub E)

By propositional logic, we have that if $E \approx F$ then $F \approx E$. Since subordination is reflexive and transitive, it follows that $\approx$ is an equivalence relation—on ways of describing the flow of all. Note that I am using the terminology of relations from thing-talk in the analyses here; I know no other way to proceed or talk. Perhaps someone will be able to formulate what we're doing here in a mass-process way of talking.

Substitution
If two categorematic words are equivalent, then they establish the same concept, or they give the same way of paying attention to the flow of all, or the equivalence codifies a correct use of words. So if one appears in a proposition, we can substitute the other for it and get a proposition with the same truth-value, for semantically they are indistinguishable.

Substitution of equivalent categorematic words If E ≈ F, and A(E) is an atomic proposition in which E appears, and A(F) is A(E) with F replacing some but not necessarily all occurrences of E, then A(F) is true iff A(E) is true.

We need this principle for atomic propositions only, as it will follow for compound ones by our use of classical propositional logic.

Commutativity of conjuncts
We have "DOG + RUN ≈ RUN + DOG", for, as we said earlier, the order is irrelevant for there being a together-mix of the two. Generally, (E + F) ≈ (F + E). Here is a rule that covers longer conjunctions, too.

Commutativity of conjuncts in a conjunction If F is a conjunction of categorematic words, and both E_1 and E_2 are conjuncts in F, and F´ is F with E_1 and E_2 replacing each other, then F ≈ F´.

This does not validate:

((CAT + LOVE) + HUMAN) ≈ ((HUMAN + LOVE) + CAT)

That's because "CAT" is not a conjunct of "(CAT + LOVE) + HUMAN"; it is a conjunct of a conjunct of that.

Subordination of a conjunction to its conjuncts
The definition of a conjunction of categorematic words says that if a conjunction is a correct description, then each conjunct is too. So "(DOG + HUMAN) sub DOG" and "(DOG + HUMAN) sub HUMAN" are true. When "SALT + WATER" correctly describes, "SALT" correctly describes, too. So "SALT + WATER sub SALT" is true. But wait, you say, there is a big difference between salt mixing with water and a dog mixing with a human. Salt and water really mix, the salt dissolves, while the dog and human remain distinguishable. To say that is to adopt the implicit metaphysics of English, where the world is made up of things and masses. Humans and dogs retain their identity: they are individuals; water and salt don't retain any identity because they have none: they are masses. From the view of the world as process-mass, human-ing is as much mass-process as water or salt. Yes, we understand the together evoked in "WATER +SALT" differently from how we understand the together evoked in "DOG + HUMAN", and that is different from the together meant in "DOG + RUN". What counts as together described by two categorematic words conjoined depends on the categorematic words that are being conjoined. We have (E + F) sub E. But we also have:

(CAT + LOVE + HUMAN) sub HUMAN

(CAT + LOVE + HUMAN) sub (CAT + LOVE)

Subordination of a conjunction to its conjuncts If E is a conjunction of categorematic words, and F is E with one of its conjuncts deleted, then E sub F.

For example, the following are true:

((CAT + LOVE) + HUMAN) sub (CAT + LOVE)

(CAT + LOVE) sub (CAT)

So by transitivity, we have:

((CAT + LOVE) + HUMAN) sub CAT

The reverse, "E sub (E + F)", does not hold. For example, "DOG sub (DOG + RUN)" is false: I can point out my window now and correctly assert "DOG" but not "DOG + RUN", for my dogs are sleeping. Nor do we have:

$\neg$ [$(E_1 + E_2)$ sub F $\rightarrow$ $((E_1$ sub F) $\wedge$ $(E_2$ sub F))]

For example, "(DOG + HUMAN) sub ANIMAL" is true, and so are "DOG sub ANIMAL" and "HUMAN sub ANIMAL".

Conjunction preserves subordination

We have "DOG sub ANIMAL". So, it seems to me, we should have:

(DOG + RUN) sub (ANIMAL + RUN)

Whatever concept of running we have that is together with dog-ing is the same concept of running mixed with animal-ing generally. This constrains our notion of "together" or "mixing" of two mass-process descriptions. I suggest we adopt the following principle.

Conjunction preserves subordination If E_1 sub E_2, then $(E_1 + F)$ sub $(E_2 + F)$.

It does not follow from the principles we've adopted that in any context in which we can describe with both "DOG" and "CAT" we can also describe with "DOG + CAT". Both "DOG" and "CAT" are correct descriptions of where the animal shelter in my town is, while "DOG + CAT" isn't. What we have, for example, is:

(1) If (DOG sub ANIMAL), then (DOG + CAT) sub ANIMAL

The consequent would be true describing a dog and cat that are sleeping touching each other or are fighting. And (1) follows from (DOG + CAT) sub (ANIMAL + CAT) and (ANIMAL + CAT) sub (ANIMAL).

Example 1 SNOW sub WHITE

WHITE sub COLOR

Therefore, SNOW sub COLOR.

Analysis Doesn't this example show that subordination isn't transitive?

No, for "SNOW sub WHITE" is false: it's not true that in any context in which we can use "SNOW" to describe correctly, we can use "WHITE" to describe correctly, as people in New York can tell you (compare this to analyzing "Snow is white" in English where we have to talk of bits of snow being white or gray). Moreover, "SNOW sub COLOR" is true: any time I point and say "SNOW" and that's a correct description, I could point and say "COLOR" and that would be a correct description (in English we'd have to say something like "All snow is colored", understanding "white(ness)" to be a color, or else say that no bit of snow is transparent).

Example 2 (GENEROUS + HATE) sub GENEROUS

GENEROUS sub VIRTUE

Therefore, (GENEROUS + HATE) sub VIRTUE

Analysis The first premise is true by the subordination of a conjunction to its conjuncts. The second seems true, too: even in English we say that generosity is a virtue, or every instance of generosity is virtuous, or any generous action is virtuous. Don't we? Would we say that an action that is both generous and hateful is virtuous? That is not an issue in syntax and semantics but in ethics. If you say no, then the second premise and the conclusion are false, for the second premise does not say that the generosity is unmixed. If you say yes, then the premises are true and the conclusion is true. The inference is valid.

Example 3 DOG sub MAMMAL

Therefore, (DOG + RUN) sub MAMMAL

Analysis Because conjunction preserves subordination, from the premise we have:

(DOG + RUN) sub (MAMMAL + RUN)

By subordination of a conjunction to its conjuncts, we have:

(MAMMAL + RUN) sub MAMMAL

So by transitivity, the conclusion follows. The inference is valid.

We have the derived principle:

(*) If (E_1 sub F), then (E_1 + E_2) sub F.

Example 4 DOG ≈ (DOG + DOG)

Analysis We have "(DOG + DOG) sub DOG" by subordination of a conjunction to its conjuncts. In the other direction we need the following principle.

Redundancy for conjunctions E sub (E + E)

We do not have the more general principle E sub (E + F), for I can point in the direction of my patio and say "DOG" and that's true, while "DOG + CAT" is false.

I'll leave as an exercise for you to show:

If (E_1 sub E_2) and (F_1 sub F_2), then [(E_1 + F_1) sub (E_2 + F_2)]

Aside: Subordination as part-whole?
The subordination relation satisfies the standard conditions for a part-whole relation: reflexivity, transitivity, and anti-symmetry.[16] But part-whole relations are about things, not masses or processes. The equivalence condition for a part-whole relation is that if object c is a part of object d and object d is a part of object c, then c and d are equal: the same thing, not equivalent or useful in place of the other. All talk of conditions on part-whole relations are about things, not how things are named. For the latter we have extensionality conditions in predicate logic.[17]

[16] See Roberto Casati and Achille Varzi, *Parts and Places* and Chapter XX of *Time and Space in Formal Logic*.
[17] But see the discussion of the predicate logic criterion of identity in *Predicate Logic*.

10 Categorematic Words as Modifiers

I point and say “doghouse”, and you agree. Seeing the world as mass-process, “HOUSE” is a correct description in that context: there is house-ing. But “DOG” need not be a correct description. My dogs could be out running. So in that context, “DOG + HOUSE” would not be a correct description. Still, the idea, the concept of dog-ing has to be involved in the description of doghouse-ing: we can understand the compound “doghouse” only if we understand “dog”. In order to have a categorematic word that is apt to use in this context, let’s adopt a new notation:

(1) HOUSE / DOG

When I assert this, it’s as if I were to assert “HOUSE” and say “think of dog-ing”. I am saying that the concept of dog-ing is needed, involved, linked somehow to house-ing. But there need not be dog-ing there, though there could be if Birta were in the doghouse. It’s hard to get this right, but it’s the same problem of how to understand “dog” in “doghouse” in English.[18] So we have:

HOUSE / DOG sub HOUSE

¬ ((HOUSE / DOG) sub DOG)

Contrast (1) with:

(2) DOG / HOUSE

This would be true if I were picking out one or more dogs that are what we call “house dogs” in English, ones that live indoors. For this we have:

DOG / HOUSE sub DOG

¬ (DOG / HOUSE sub HOUSE)

Contrast (1) and (2) with:

(3) DOG + HOUSE

I could have used this correctly when a friend’s dog came into my house and laid down. For this we have:

DOG + HOUSE sub DOG

DOG + HOUSE sub HOUSE

The choice of which of (1), (2), or (3) to use depends on which of the categorematic words in the composite the composite is subordinate to.

Now suppose I want to describe a context in which my dog Birta is hungry. I can use:

DOG + HUNGER

[18] See Appendix 1, “Compound Nouns and Meaning”.

That's because in any context in which this is true, both "DOG" and "HUNGER" are correct descriptions, which isn't the case if I use "DOG / HUNGER" or "HUNGER / DOG".

Modified categorematic words If E and F are categorematic words, then E / F is a *modified categorematic word.* The word F *modifies* the word E; it is the *modifier.*

The word E / F is a good/accurate/correct description in a context if E is a good/accurate/correct description in a way modified by our understanding of F.

The general principle illustrated in the examples is the following.

Subordination of a modified categorematic word (E / F) sub E

Though "(E / F) sub F" can be false, as in the examples above, it need not be. For example, "HOUSE / WOOD sub WOOD" is true, at least in a model of how we ordinarily live. We can adopt a meaning axiom in cases for which ¬ ((E / F) sub F) holds, for example, "¬ (HOUSE / DOG sub DOG)".

Now consider:

HOUSE / HOUSE

Using "HOUSE" to modify itself neither adds nor subtracts anything. It's the same concept as given by "HOUSE". That is,

HOUSE / HOUSE ≈ HOUSE

Redundancy for modifications (E / E) ≈ E

Example 1 Toy bears and bear toys.

Analysis In English I can point to a shop window and say "toy bear": that's a toy resembling or somehow meant to evoke the idea of a bear. At the zoo I can point to a bear playing with a tire hung from a branch and say "bear toy": that's a toy meant for bears to play with. But in our mass-process language we have only one word to use in both contexts:

TOY / BEAR

To distinguish the two contexts, we could use an additional description for the bear playing with the tire swing:

(TOY / BEAR) + (BEAR + TOY)

I can't think of a context in which it would be correct to use "BEAR / TOY".

Example 2 Cartoon cat.

Analysis Suppose I'm watching TV and I call to you and point to the screen and say "cartoon cat". In that context, "CARTOON" is a correct description, but "CAT" is not. Unless, that is, you think that cartoon cat-ing is just another kind of cat-ing. That does not seem a good way to go, no more than in our thing-conception of the world we should populate the universe with not only siamese cats and calico cats but also cartoon cats and imaginary cats. So we can't use "CARTOON + CAT" to describe in that context. Yet the idea, the concept of cat-ing has to be involved in the description in some way. We use "CARTOON / CAT".

Example 3 Fake dog.

Analysis I have a small purple hand puppet called "Ralph" that looks like a dog. There, pointing, is fake-ing, imitation-ing if you like, but not dog-ing. So to describe when I point to it, I can use:

FAKE / DOG

There is fake-ing in the style of, meant to suggest, looking like, resembling in some way dog-ing. But there is not fake-ing and dog-ing mixed, for which we would use:

DOG + FAKE

This would have been a correct description of the two dogs Birta and Buddy I had when I fed them. Birta would eat hurriedly then run off a little way and start barking as if there were something in that direction. Buddy, who was bigger and dominant, would then run there, too, and start barking, and Birta would run back to eat Buddy's food while he was barking. In contrast, to correctly describe what we in thing-talk would call a dog dressed up in a Santa Claus costume we can use:

DOG / FAKE

Example 4 Imaginary dog.

Analysis A fellow talks when there's no one around. He says he's talking to a dog, but there's no dog there. Pressed, he says uncomfortably that he's talking to an imaginary dog. An imaginary dog is not a dog. An imaginary dog would be outside space and time, it seems. But we need not talk of an imaginary dog in that context but only of imagine-ing somehow linked to the concept of dog-ing, resembling in some way dog-ing, for which we can use:

IMAGINE / DOG

I'll let you supply contexts to distinguish the correct use of this from contexts in which "DOG / IMAGINE" or "DOG + IMAGINE" are correct.

Example 5 Fake unicorn.

Analysis Consider:

FAKE / UNICORN

This can be true when I point to a stuffed toy that looks like what we imagine a unicorn to be. Not only is "UNICORN" not a correct description in this context, it is not a correct description in any context in the world as we know it. Yet it is useful as a modifier, for it does elicit a conception.

Example 6 STORY / PEGASUS

Analysis This would be a correct description of someone telling a story—and here we get into a mess in English. A story about Pegasus? What does "about Pegasus" mean if "Pegasus" does not pick out, refer to, stand for any thing, at least no thing that exists? How to talk coherently and reason about fiction is difficult in out thing language; I've worked out some ways to do that in *The Internal Structure of Predicates and Names.* But here we can have a name such as "PEGASUS" that's true in no context (in the way the world is and has been). Yet, like "UNICORN", it establishes a concept. In particular, "PEGASUS sub HORSE" is true, and "PEGASUS sub WING" is true. And "STORY / PEGASUS" can be true in a context. There is no difference in how "PEGASUS" and "ZOE" can be used. It's just that "PEGASUS" by itself never describes correctly, while in some contexts "ZOE" does describe correctly. It's really that easy in our mass-process language to have fiction talk. Or "TALK / FICTION".

Example 7 Competent teaching.

Analysis Suppose a mother goes to watch her child's 4th grade arithmetic class; she says later that she saw competent teaching. Should we say that she saw both teaching and competence? Can there be competence-ing absent a comparison? I'm not sure. Is competence the same for teaching and for auto repair? Is it the same for ski-ing? Can we take "COMPETENCE" as a categorematic word? There are going to be lots of examples like this that will stop us. It's not just that we have to think of the concept of competence; after all, "competent" is defined in the dictionary without reference to teaching, auto repair, or ski-ing. No, the issue is how to think of competence in a mass-process language, and for that it would be better to see how that concept is used in ordinary mass-process languages.

Modified categorematic can be used in the same way as other categorematic words
A modified categorematic word can be used in a conjunction. For example, to describe when my donkey is carrying around my dog puppet on her back I can use:

DONKEY + (FAKE / DOG)

A modified categorematic word can be modified further, as with:

(HOUSE / DOG) / CASTLE

This would be correct were I to point at what in English we'd say is a doghouse

that is shaped like a castle. And we can use a modified categorematic word to modify a categorematic word, as with:

(HOUSE / DOG) / (CASTLE / FAIRY)

Modifiers that aren't categorematic words

To say that there is very loud barking is to say that not only is the barking loud, but compared to other loud barking it is loud. It seems that "very" is used only for a comparison with a base. How could we say that (pointing) there is very-ing? It doesn't make sense to use "very" as a base categorematic word.

Consider also "nearly". To say that Birta is nearly barking is clear enough. But how could we use "nearly" as a base categorematic word—there (pointing) is "nearly-ing"?

Consider also "almost". To say that this pond is almost frozen is clear enough. But how could we use "almost" as a base categorematic word—there is "almost-ing"?

There are some adjectives and adverbs from English that we would like to use as modifiers of categorematic words but not as categorematic words themselves. The categorematic words are the concept words of our language. They are categorematic. The words "very", "nearly", "almost", and others are syncategorematic: they have meaning only when used with a categorematic word. But they are not logical words like our formalizations of "and" or "not". We can say they are non-logical-syncategorematic.

How could we use those in our language? We can write them in lower case to distinguish them from the base categorematic words:

very
almost
nearly

We could then use them as modifiers with the same slash notation:

BARK / almost

RED / nearly

MEOW / (LOUD / very)

In this way, perhaps, we could also have other modifiers that we wouldn't want to take as categorematic words, for example, "fast" or "big". Then we could say "RUN / fast". These would set up comparisons: "RUN / fast" is true if the running I am pointing to is fast in comparison to other running. These would also be syncategorematic, pulling their meaning by attachment to a categorematic word, that is, to a categorematic word.

If we were to adopt such modifiers into our language, we would have to divide them into restrictors and negators as we did modifiers of predicates in Volume 1. We have "MEOW / (LOUD / very) sub (MEOW / (LOUD)" but "¬ (BARK / almost) sub BARK". That would complicate the presentation of the language and logic

considerably and, I fear, obscure the more basic points. Moreover, we should consider how or whether such modifiers are used in ordinary mass-process languages as a guide. So I will leave that project to others.

Substitution of equivalents?
Some, it seems, take as true:

(4) (MIND + HUMAN) ≈ (BODY + HUMAN)

Yet if we were to adopt "fast" as a modifier, as suggested above, both of the following would be false:

((MIND + HUMAN)/fast) sub ((BODY + HUMAN)/fast)

((BODY + HUMAN)/fast) sub ((MIND + HUMAN)/fast)

An olympic sprinter could be as mentally slow as a president, and a university professor could walk barely fast enough to get to her classroom in time. Doesn't this show that the principle of substitution of equivalent categorematic words can fail?

The problem here is not with substitution but with (4). I can understand how someone could believe, and perhaps it is true, that in any context in which it is correct to assert "MIND + HUMAN" it is correct to assert "BODY + HUMAN", and vice-versa. But that, recall, is a consequence of assuming (4), not the basis for the truth of (4). I do not see how even a committed materialist could say that the concept of "MIND + HUMAN" is the same as the concept of "BODY + HUMAN", or that they are equivalent ways of paying attention in the flow of all, or that (4) codifies a correct use of words.

11 A Formal Logic of Mass-Process and Subordination

The formal language

Vocabulary

base categorematic word symbols $B_1, B_2, \ldots, B_n, \ldots$

subordination symbol sub

categorematic word conjoiner +

propositional connectives $\neg, \rightarrow, \wedge, \vee$

Punctuation

parentheses ( )

slash /

Categorematic words

i. If B is a base categorematic word symbol, then (B) is a formal categorematic word of degree 1.

ii. If $E_1, \ldots, E_r$ are formal categorematic words where $r \geq 2$ and the maximum of the degrees of $E_1, \ldots, E_r$ is n, then $(E_1 + \ldots + E_r)$ is a formal categorematic word of degree $n + 1$. It is a *conjunction of* formal categorematic words. Each of $E_1, \ldots, E_r$ is a *conjunct* of it.

iii. If E and F are formal categorematic words, and the maximum of the degrees of E and F is n, then E / F is a formal categorematic word of degree $n + 1$. It is a *modified* formal categorematic word. F is the *modifier* and E is the word *modified.*

iv. A concatenation of symbols is a formal categorematic word iff for some n it is a formal categorematic word of degree n.

If E is a formal categorematic word of degree ≥ 2, then E is a *complex* formal categorematic word. I'll let you show that there is one and only one way to parse each formal categorematic word.

Well-formed formulas (*wffs*)

i. If E and F are formal categorematic words, then (E sub F) is a wff of length 1.

ii. If A is a wff of length n, then $(\neg A)$ is a wff of length $n + 1$.

iii. If A and B are wffs and the maximum of the lengths of A and B is n, then each of $(A \rightarrow B)$, $(A \wedge B)$, $(A \vee B)$ is a wff of length $n + 1$.

iv. A concatenation of symbols is a *wff* iff it is a wff of length *n*, for some $n \geq 1$.

A wff of length 1 is *atomic.* All other wffs are *compound.*

The unique readability of wffs follows as for classical propositional logic.

We use the following definition:

$E \approx F \equiv_{Def} (E \text{ sub } F) \wedge (F \text{ sub } E)$

We adopt from propositional logic the usual conventions on informally deleting parentheses. We also informally delete parentheses around a base categorematic word when that isn't likely to lead to confusion.

Realizations and semi-formal languages

A categorematic word of our "ordinary" language is *simple* iff it contains no proper part that we could formalize as a categorematic word, some combination of those with + and /, or a propositional connective.

A *realization* of the formal language is an assignment of simple categorematic words to the base categorematic symbols. The resulting wffs constitute a *semi-formal language*. We do not assume that there is just one realization where "all" categorematic words are used: we don't need to use all ways of dividing the flow of all to have any talk at all. I will write "$\upsilon(E \approx F) = \top$" as an abbreviation of: "$\upsilon(E \text{ sub } F) = \top$ and $\upsilon(F \text{ sub } E) = \top$".

Semantics

The notion of a model is as for classical propositional logic: a realization plus an assignment of truth-values to the atomic propositions, with the usual evaluation of compound wffs. In addition, we require that the evaluation of atomic wffs satisfies the principles for the subordination relation we adopted in the preceding chapters.

Explicitly, given a semi-formal language , a *valuation* υ is an assignment of truth-values to the atomic propositions of it. The valuation satisfies the following conditions respecting the internal structure of atomic propositions.

Reflexivity of subordination
$\upsilon(E \text{ sub } E) = \top$

Transitivity of subordination
If $\upsilon(E_1 \text{ sub } E_2) = \top$ and $\upsilon(E_2 \text{ sub } E_3) = \top$, then $\upsilon(E_1 \text{ sub } E_3) = \top$.

Substitution of equivalent categorematic words If $\upsilon(E \approx F) = \top$, and A(E) is an atomic proposition in which E appears, and A(F) is A(E) with F replacing some but not necessarily all occurrences of E, then $\upsilon(A(F)) = \top$ iff $\upsilon(A(E)) = \top$.

Commutativity of conjuncts in a conjunction If F is a conjunction of

categorematic words, and both E_1 and E_2 are conjuncts in F, and F´ is F with E_1 and E_2 replacing each other, then $\cup(F \approx F') = \top$.

Subordination of a conjunction to its conjuncts
If E is a conjunction of categorematic words, and F is E with one or more of its conjuncts deleted, then $\cup$(E sub F) = $\top$.

Conjunction preserves subordination
If $\cup(E_1 \text{ sub } E_2) = \top$, then $\cup((E_1 + F) \text{ sub } (E_2 + F)) = \top$.

Redundancy for conjunctions
$\cup$(E sub (E + E)) = $\top$

Subordination of a modified categorematic word
$\cup((E/F) \text{ sub } E) = \top$

Redundancy for modifications
$\cup((E/E) \approx E) = \top$

A *model* is the semi-formal language, a valuation, and the extension of the valuation to all compound wffs by the truth-tables:

$\cup(\neg A) = \top$ iff $\cup(A) = \mathsf{F}$

$\cup(A \rightarrow B) = \top$ iff $\cup(A) = \mathsf{F}$ or $\cup(B) = \top$

$\cup(A \wedge B) = \top$ iff $\cup(A) = \top$ and $\cup(B) = \top$

$\cup(A \wedge B) = \top$ iff $\cup(A) = \top$ or $\cup(B) = \top$

This is the *base logic of mass-process and subordination,* **MP Sub**.

We do not assume that there is only one model with the correct assignment of truth-values to the atomic propositions. We might want to investigate the consequences of whether "WHITE sub COLOR" is true by considering a model in which it is true and a model in which it is false.

Axiomatization

Propositional axioms

The axiom schemes of classical propositional logic (Chapter 6 of Volume 0), where A, B, and C are any wffs of the formal language.

$\forall \ldots (\neg A \rightarrow (A \rightarrow B))$

$\forall \ldots (B \rightarrow (A \rightarrow B))$

$\forall \ldots ((A \rightarrow B) \rightarrow ((\neg A \rightarrow B) \rightarrow B))$

$\forall \ldots ((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)))$

$\forall \ldots (A \rightarrow (B \rightarrow (A \wedge B)))$

$\forall \ldots ((A \wedge B) \rightarrow A)$

$\forall \ldots ((A \wedge B) \rightarrow B)$

$\forall \ldots (A \rightarrow (A \vee B))$

$\forall \ldots (B \rightarrow (A \vee B))$

$\forall \ldots ((A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow ((A \vee B) \rightarrow C)))$

Subordination axioms

Reflexivity of subordination
E sub E

Transitivity of subordination
$((E_1 \text{ sub } E_2) \wedge (E_2 \text{ sub } E_3)) \rightarrow (E_1 \text{ sub } E_3)$

Substitution of equivalent categorematic words
$E \approx F \rightarrow (A(F) \leftrightarrow A(E))$
where A(E) is an atomic proposition in which E appears, and A(F) is A(E) with F replacing some but not necessarily all occurrences of E.

Commutativity of conjuncts in a conjunction
$F \approx F'$
where F is a conjunction of categorematic words, and both E_1 and E_2 are conjuncts in F, and F′ is F with E_1 and E_2 replacing each other.

Subordination of a conjunction to its conjuncts
E sub F
where E is a conjunction of categorematic words, and F is E with one of its conjuncts deleted.

Conjunction preserves subordination
$(E_1 \text{ sub } E_2) \rightarrow [(F + E_1) \text{ sub } (F + E_2)]$

Redundancy for conjunctions
E sub (E + E)

Subordination of a modified categorematic word
(E / F) sub E

Redundancy for modifications
$(E / E) \approx E$

Rule *modus ponens* $\dfrac{A,\ A \rightarrow B}{B}$

The definitions of theorem and valid formal inference are the usual ones (Volume 0).

Each axiom of the formal system is a scheme of tautologies. For the propositional logic axioms this was shown in Volume 0. For the subordination axioms this follows by the conditions on the valuations of atomic wffs. Since the single rule preserves truth in a model, each of the theorems of our system is a scheme of tautologies. And every formally valid inference is a scheme of valid inferences. Moreover, by the constructive proof of completeness of classical propositional logic[19], this logic is complete, relative to these formal semantics.

Our reasons for accepting the conditions on valuations on atomic wffs are not reflected in this formal system. They come from the ideas of meaning and truth for our mass-process language developed in the preceding chapters. They are based on experience, but not the experiences of our ordinary lives, for those are grounded in and shaped by our thing-language.[20] Even establishing a formal language I have had to use methods and concepts of thing-languages, giving inductive definitions. I cannot help but do this, for what we are doing here is looking at the idea of the world as process from the outside, as an anthropologist would look at a very different culture, trying to understand it while nonetheless filtering it through his or her own culture and language. I hope to have led you to begin talking a bit in this mass-process way so you can have some experiences in that way of being in the world. A monolingual speaker of a mass-process language has a wealth of experience in seeing and living the world as mass-process. But to convey our ideas to him or her, one of us must speak the other's language, and then perhaps we can ask whether our work is apt as an abstraction from what he or she knows—if abstraction is even an intelligible idea for a person raised in that language and culture.

We have no reason to think that if a realization of a wff of this language is a tautology relative to our informal semantics, our view of the world as process-mass, then the wff is a theorem of the formal system. The best and to my knowledge the only way to ensure that we have sufficient axioms for every informal tautology to be a theorem is to regularize the idea of context, making clear(er) the notion of model for this logic by formalizing and restricting what we mean by a context. Then we can try to prove a completeness theorem, either being successful or finding new axioms that we need.[21] How can we make the notion of context for a mass-process language precise?

Most naturally for us as inheritors of the Western tradition of logic and reasoning is to formalize context in terms of time and location, which we'll do in the next section. But that will involve treating times and locations as things, so we will be viewing mass-process through our own language and community.

In the last section of this text on reasoning about the world as process, we'll see how to regularize the idea of context with a minimal idea of time, thinking only

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[19] See Carnielli's and my *Computability*.

[20] Note the use of the word "experience" as if that were a thing, whereas "EXPERIENCE" is most certainly a mass-process word.

[21] See my "Possibilities and Valid Inferences".

in terms of before and after, which will be more compatible with the idea of the world as process-mass.

These are only two ways we can formalize the idea of a context in which a mass-process word is or is not a good/accurate/correct description. We could, perhaps, mark contexts by pointing, or by touch, or, if our noses were as sensitive as those of dogs, by smells. But that is another story.

Contexts Given by Times and Locations

12 Times and Locations

The categorematic word “RAIN” is neither true nor false without a context. One way to provide a context is to say the when and where the word is meant to describe:

RAIN now, there

RAIN 3 p.m. September 12, 2018, Socorro

These are true or false. Can we use time and space to establish contexts in this way? To do so, it seems, would be to talk of times and places as things.

Time is process-mass. Time flows, time runs, time is a river. Space is process-mass. Space is all-encompassing, every part of space is space. Indeed, time and space are archetypal process-mass.

We do not count time; it is not a thing, nor is it composed of things. We measure time, we divide it up by measuring. We have standard measures. The first were days, then periods between moons, and then periods between solstices. Now we have more ways, more precise for talking about parts of the process-mass of time. We talk about hours, minutes, seconds, nanoseconds. We talk of weeks, months, years, decades, centuries. We measure with these relative to a fixed part of time, whether that be now or what we call 0 BC/AD, or when Juney died. Words like “yesterday” and “12:42 p.m. April 20, 2010” are ways we use to focus attention on some of the process of time. But we treat them as if they pick out things. This is not to deny their process-mass nature but rather to pay attention to measures of time in a very restricted way. No less than Zoe-ing or dog-ing, there was yesterday-ing, though we choose not to look at it that way in our reasoning.

We do not count space; it is not a thing, nor is it composed of things. We measure space, we divide it up by measuring. The oldest measures used rocks, trees, and other parts of a landscape to mark off some area. Later cubits, or feet, or hand-lengths were used. Now we have more ways, more precise for talking about the process-mass of space. We use meters, cubic meters, light-years to divide up space with reference to some fixed location. We talk of the parcel of land described on a survey, we talk of the space enclosed by the Al-Aqsa Mosque in Jerusalem. None of these is a thing, for some of a mass is still mass, some of a process is still process. The path from my house to the river is mass-process, but I conceive of it as a thing in order to talk about some of the flux of space, to allow you, too, to focus your attention by dividing, measuring in the same way.

There is a great difference between how we measure time and how we measure space. We have fixed time markers in our language, each marking an interval, a mass of time within the mass of all time. We use these to talk about times. But we have no standard measures we use in our ordinary speech for space, a mass of space within the mass of all space. We cannot use a coordinate system to pick out regions

of space generally because, even discounting the difficulty of reconciling points of space denoted by numbers with the process of space, we have no idea how to give coordinates for the boundary of the corral at my ranch. What dividing we do of space is via talk of what is true about some of space, as when we describe a location as one in which Spot is barking and Dick is yelling, or via pointing and marking, as when we put up survey stakes. We use many ways to divide space, though no standard ones.

We have ways of talking about the archetypal masses of time and space as divided into parts. To the extent we believe such ways of measuring-dividing—treating times and locations as things—are compatible with a view of the world as process-mass, we can use them to devise a way to make descriptions of the world with our categorematic words. Or at least in the hope of making some clearer bridge from our view of the world as made up of things and the world as process-mass, let's treat the results of measuring and dividing time and space as things. Viewed as things we can reason about them with classical predicate logic.

From Volume 2, we have the pure logic of space and time that we can use here. We have first a two-sorted language for predicate logic.

Vocabulary for times and locations as things

time name symbols $\mathrm{b}_0, \mathrm{b}_1, \ldots$
time variables $t_0, t_1, \ldots$
} *time terms*

location name symbols $\mathrm{e}_0, \mathrm{e}_1, \ldots$
location variables $l_0, l_1, \ldots$
} *location terms*

time order predicate $— <_{\text{time}} —$

time part predicate $\mathrm{W}_{\text{time}}(—, —)$

time equality predicate $\equiv_{\text{time}}$

location part predicate $\mathrm{W}_{\text{location}}(—, —)$

location equality predicate $\equiv_{\text{location}}$

quantifiers $\forall, \exists$

We use $t, w, t', w', w_0, w_1, \ldots$ as meta-variables ranging over time terms, and $l, l', p, p', p_0, p_1, \ldots$ as meta-variables ranging over location terms.

We can have different universes of time for a model, corresponding to different ways of staking out parts of the mass-process of time. We can have different universes of locations even more naturally, for we have no standard names for parts. We stake out parts of space with our descriptions, and we talk of those parts as things, and we find that what we are reasoning about in adopting a universe of locations are the ways we describe parts of that mass.

For our logic, we have the axioms of propositional logic and the usual axioms governing quantifiers in a two-sorted predicate logic, for example $\forall t\, \forall w\, \mathrm{A} \rightarrow\ \forall w\, \forall t\, \mathrm{A}$. We also assume that how we refer to times and things does not matter: our use of variables and names for them is extensional. I'll use t for t_1, w for t_2, t' for t_3, w' for t_4, l for l_1, p for l_2, and l' for l_2.

Axioms for equality and extensionality

$\forall t\, (t \equiv_{\text{time}} t)$

$\forall \ldots\, \forall t\, \forall w\, (t \equiv_{\text{time}} w \rightarrow\ (\mathrm{A}(t) \rightarrow \mathrm{A}(w/t)))$
where A is atomic and w replaces some
but not necessarily all occurrences of t in A

$\forall l\, (l \equiv_{\text{location}} l)$

$\forall \ldots\, \forall l\, \forall p\, (l \equiv_{\text{location}} p \rightarrow\ (\mathrm{A}(l) \rightarrow \mathrm{A}(p/l)))$
where A is atomic and p replaces some
but not necessarily all occurrences of l in A

In Volume 2, we adopted assumptions about the nature of time and space that models must satisfy. The motivations/justifications for those, we were careful to note there, are compatible with viewing times and locations as parts picked out from the mass-process of time and space. So we adopt them here.

Axioms for time

W_{time} *is a part-whole relation*

$\forall t\, \mathrm{W}_{\text{time}}(t\, ,\, t)$

$\forall t\, \forall w\ (\mathrm{W}_{\text{time}}(t\, ,\, w) \wedge\ \mathrm{W}_{\text{time}}(w\, ,\, t) \rightarrow\ (t \equiv_{\text{time}} w))$

$\forall t\, \forall w\, \forall t'\ (\mathrm{W}_{\text{time}}(t,\, w) \wedge\ \mathrm{W}_{\text{time}}(w,\, t') \rightarrow\ \mathrm{W}_{\text{time}}(t,\, t'))$

Parts determine times

$\forall t\, \forall w\ ((t \equiv_{\text{time}} w)\ \leftrightarrow\ \forall t'\, (\mathrm{W}_{\text{time}}(t'\, ,\, t) \leftrightarrow \mathrm{W}_{\text{time}}(t',\, w)))$

$<_{\text{time}}$ *determines an ordering*

$\forall t\, \neg\, (t <_{\text{time}} t)$

$\forall t\, \forall w\, \forall t'\ ((t <_{\text{time}} w) \wedge\ (w <_{\text{time}} t') \rightarrow\ (t <_{\text{time}} t'))$

Parts and wholes are unrelated in the ordering

$\forall t\, \forall w\ (\mathrm{W}_{\text{time}}(t,\, w) \rightarrow\ \neg\, (t <_{\text{time}} w) \wedge\ \neg\, (w <_{\text{time}} t))$

Parts of times are related to other times in the ordering as the whole is related

$\forall t\, \forall w\, \forall t'\ (\mathrm{W}_{\text{time}}(t,\, w) \wedge\ (w <_{\text{time}} t')) \rightarrow\ (t <_{\text{time}} t'))$

$\forall t\, \forall w\, \forall t'\ (\mathrm{W}_{\text{time}}(t,\, w) \wedge\ (t' <_{\text{time}} w)) \rightarrow\ (t' <_{\text{time}} t))$

Times are intervals

1. $\forall t\, \forall w\, \forall t'\ (\mathrm{W}_{\mathrm{time}}(t, t') \wedge \mathrm{W}_{\mathrm{time}}(w, t') \rightarrow \mathrm{W}_{\mathrm{time}}(w, t) \vee \mathrm{W}_{\mathrm{time}}(t, w) \vee \mathrm{X}_{<}(t, w) \vee \mathrm{X}_{<}(w, t) \vee (t <_{\mathrm{time}} w) \vee (w <_{\mathrm{time}} t)$

2. $\forall t\, \forall w\, \forall t'\ (\mathrm{W}_{\mathrm{time}}(w, t) \wedge \mathrm{W}_{\mathrm{time}}(t', t) \wedge (w <_{\mathrm{time}} t') \rightarrow \forall t_4\, ((w <_{\mathrm{time}} w' <_{\mathrm{time}} t') \rightarrow \mathrm{W}_{\mathrm{time}}(w', t)))$

Overlapping times are not related in the ordering

$\forall t\, \forall w\ (\mathrm{X}_{<}(t, w) \rightarrow (\neg (t <_{\mathrm{time}} w) \wedge \neg (w <_{\mathrm{time}} t)))$

$\mathrm{X}_{<}(t_1, w)$ is a defined predicate that is true for $\mathbf{t}$ and $\boldsymbol{w}$ as references for t and w iff there is some time in both $\mathbf{t}$ and $\boldsymbol{w}$ and there is some time in $\mathbf{t}_1$ that is before every time in $\boldsymbol{w}$.

Axioms for locations

$\mathrm{W}_{\mathrm{time}}$ *is a part-whole relation*

$\forall l\, \mathrm{W}_{\mathrm{time}}(l, l)$

$\forall l\, \forall p\ (\mathrm{W}_{\mathrm{time}}(l, p) \wedge \mathrm{W}_{\mathrm{time}}(p, l) \rightarrow (l \equiv_{\mathrm{location}} p))$

$\forall l\, \forall p\, \forall l_3\ (\mathrm{W}_{\mathrm{time}}(l, p) \wedge \mathrm{W}_{\mathrm{time}}(l, l_3) \rightarrow \mathrm{W}_{\mathrm{time}}(l, l_3))$

Parts determine locations

$\forall l\, \forall p\ ((l \equiv_{\mathrm{location}} p) \leftrightarrow \forall l_3\, (\mathrm{W}_{\mathrm{location}}(p', l) \leftrightarrow \mathrm{W}_{\mathrm{location}}(p', p)))$

Note that the part-whole relations are for a time as part of another time, a location as part of another location, not for a time as part of "all" time, a location as part of "all" space.

Should we adopt more assumptions to reflect our view of time and space as mass, perhaps requiring that there is no smallest time or smallest location? We may wish to restrict our attention in our reasoning—that is, in a particular model—to only some of the flow of all, and doing so we might want to allow for smallest times or smallest locations. So let us adopt only these minimal assumptions from Volume 2, allowing others to be imposed for particular models.

We have a two-sorted pure logic of space and time. Now let's consider how to use times and locations to establish contexts for the truth or falsity of categorematic words.

13 Times and Locations for Context

We have categorematic words, as defined and conceived of in the logic of mass-process and subordination. We can give a context for, say, "RAIN" with a time and place. Then "RAIN" is either true or false at that time in that place.

This is compatible with treating "RAIN" as a predicate of a time and place—but not, as in the logic of physical things of Volume 2, also of an individual or individuals. We can mark "RAIN" and any other categorematic word as a predicate in this way. For example:

RAIN ($—_{time}$, $—_{location}$)

DOG ($—_{time}$, $—_{location}$)

(DOG / HOUSE) ($—_{time}$, $—_{location}$)

(CAT + MEOW) ($—_{time}$, $—_{location}$)

Each will be an atomic predicate. Note that an atomic predicate can have internal structure.

The time-marked blank is meant to be filled with a time term, and the location-marked blank with a location term in order to have a well-formed formula. So the following are well-formed formulas:

DOG ($—_{time}$, $—_{location}$) (January 6 1947, Omaha)

(DOG / HOUSE) ($—_{time}$, $—_{location}$) (September 13 2018, corral)

(CAT + MEOW) ($—_{time}$, $—_{location}$) (12:15 pm May 6 2011, Socorro)

RAIN ($—_{time}$, $—_{location}$) (t, l)

We take as primitive whether these are true or false in a model (with references supplied for the variables) just as in classical predicate logic we take "— is a dog" as true or false of Birta. In both cases we are building on an intuition and much motivation and discussion of when such an expression is true. This is not to say that I think that the idea of a categorematic word being a good/accurate/correct description at a time in a place is clear. It is certainly not familiar. But I hope to have led you in our work on the logic of mass-process and subordination to see how to use the idea well enough to start our investigations. We hope to come to a better understanding as we use it.

As an example, consider:

DOG ($—_{time}$, $—_{location}$) (February 6 2010, l)
where l refers to a location of my ranch Dogshine

This is true iff it is accurate to describe the part of flow at that time and in that place

with "DOG" . In thing-talk we'd say it's true if there was a dog or there were dogs at that time and place. But "DOG" here is not a name, nor does it pick out anything. Words like "RUN", "MUD", "DOG + PLAY", "DOG/HOUSE" used as categorematic words do not denote, they do not refer to anything in the world. They are used to describe some of the flux of all.

But doesn't invoking time and space for establishing a context mean that we are constrained to talk only of the material world? There is no conception of material versus abstract in the view of the world as process-mass. There is no conception of the physical. There is only the flow of all, described in various ways. The flow of all is not in time and space but time and space are the web and woof of the flow of all.

What assumptions shall we make about truth of a categorematic word in relation to times and locations? Consider again,

(1) DOG $(\text{—}_{\text{time}}, \text{—}_{\text{location}})$ (February 6 2010, l)
where l refers to a location of my ranch Dogshine

If this is true, then "DOG $(\text{—}_{\text{time}}, \text{—}_{\text{location}})$" is true then of any location that encloses my ranch Dogshine, any location of which the location specified for l is a part, for example, the state of New Mexico. We adopt just as for the logic of physical things an assumption of the outward closure of truth for locations. I'll give the principles syntactically because they're easier to read.

Outward closure of truth for locations

$$\forall\, t\, \forall l\, [\, \mathrm{E}(\text{—}_{\text{time}}, \text{—}_{\text{location}})\, (t, l) \rightarrow$$
$$(\forall p\, (\mathrm{W}_{\text{location}}(l, p) \rightarrow \mathrm{E}(\text{—}_{\text{time}}, \text{—}_{\text{location}})\, (t, p))\,]$$

If (1) is true, then it is true of any "smaller" time, too, that is any time that is within February 6 2010. And if it is true of every time within February 6 2010, it is true at February 6 2010. We adopt with the same motivation as for the logic of physical things an assumption of the downward and upward closure of truth in time.

Downward and upward closure of truth in time at a location

$$\forall\, t\, \forall l\, [\, \mathrm{E}(t, l) \leftrightarrow \forall w\ (\mathrm{W}_{\text{time}}(w\, , t) \rightarrow \mathrm{E}(w, l))\,]$$

We have the categorematic words from the logic of mass-process and subordination. Should we incorporate all of that logic by including reasoning using "sub" for the subordination relation?

Subordination is about relating conceptions that categorematic words elicit, or it can be understood as relating ways of paying attention to the flow of all, or we can understand it as codifying the correct use of words. It is a consequence of a subordination proposition such as "DOG sub ANIMAL" that in every context in

which "DOG" is true, so, too, is "ANIMAL". But that is not all there is to subordination. In a model meant to codify what (we believe) is true of our world now and in the past, we would have that in every context in which "UNICORN" is true, "AUTOMOBILE" is true, for "UNICORN" is true in no context. Yet we want "UNICORN sub AUTOMOBILE" false and "UNICORN sub ANIMAL" true, for those codify the conceptual relation or they mark correct uses of those words. We could even have a model with so few locations and times that the following could be true:

$\forall\ t\ \forall l\ (\ (\text{DOG}\ (\text{—}_{\text{time}}, \text{—}_{\text{location}})\ (t, l) \rightarrow\ \text{HUMAN}\ (\text{—}_{\text{time}}, \text{—}_{\text{location}})\ (t, l)\ )$

Yet, unless we are very committed mass-process nominalists, we won't want to say for that model "DOG sub HUMAN" must be true. We need to retain the logic of categorematic words and subordination, while linking it to truth in context with the following.

Subordination and truth in a context

$(\text{E sub F}) \rightarrow\ \forall t\ \forall l\ (\text{E}\ (\text{—}_{\text{time}}, \text{—}_{\text{location}})\ (t, l) \rightarrow\ \text{F}\ (\text{—}_{\text{time}}, \text{—}_{\text{location}})\ (t, l)\ )$

I'm going to assume that what I've described is enough for us to continue our discussions, leaving a full formal presentation for Chapter 22 when we have added more to our language and semantic assumptions.

Aside: *Water everywhere*

Some people who wish to analyze talk of masses in ordinary English say that "water" is a mass-term which refers to all water at all times and places.[1] The water in the pond in my patio is part of that water, and similarly for other masses. But this seems too broad, indeed, a confusion of the term "water" being true of various times and places with the wish to have something corresponding to it, almost an abstract entity, which is what is at all those times and places. We do not say that my dog Birta is part of all dogs that are at all times and places; we may, if we are platonists, say there is a thing that is the collection of all dogs at all times and places and Birta is part of that, but to do so is a great way from our ordinary talk and demands a great deal of metaphysics beyond what most of us find plausible or necessary to explain the truth of what we say. In the view of the world as flux, "water" is a categorematic word which we understand to be true or false of particular times and locations. There is no water we can refer to that is of all times and places, for there is no largest time nor largest location we can talk about: to assume there is would be to deny the mass-process nature of time and location. To say that this water here and now is part of all water is to say no more than that "$\text{WATER}(\text{—}_{\text{time}}, \text{—}_{\text{location}})\ (t, l)$" is true of here and now.[2]

[1] Willard Van Orman Quine says in *Word and Object:*

> There is no reason to boggle at water as a single though scattered object, the aqueous part of the world. p. 98

[2] Harry C. Bunt in *Mass Terms and Model-Theoretic Semantics* presents a theory of parts for masses which is based on an atemporal, alocational conception of mass.

14 Examples of Formalizing: Predications

We are not formalizing reasoning from an ordinary language here. We are devising a way to talk about the world from a different perspective than our ordinary language allows. But what we mean by "the world" is not clear. What we think of as the world is surely shaped by the language we use, as I hope to have led you to see in these chapters. Perhaps what we are trying to do here is create a new world, not denying that there is something "out there" but insisting that how we see is not a trivial addition to our sensory inputs or a clear lens to see precisely what can be seen in only one way. Still, I have no choice but to give examples in English, either as propositions that we can try to re-interpret or as descriptions of an experience we might have that we can try to cast in the view of the world as flux.

In order to make the examples and discussions easier to read, I will often delete the blanks in a predicate, since in all cases there will be two blanks: one for time and one for location. So instead of "(RAIN) ($—_{time}$, $—_{location}$) (t, l)" I'll write "RAIN (t, l)". I'll also assume that we have a name "now" in our semi-formal language meant to pick out the present of the model.

Example 1 Birta is a dog.

Analysis This is thing-talk. Logicians call it a predication.

If what is meant is that Birta has some property, some quality, is a dog in some essential way, then we can formalize the example as:

(a) BIRTA sub DOG

Birta-ing is dog-ing. Not now, not always, but in the relation of those ways of describing, a relation of concepts, or as a correct use of those words.

The proposition (a) is atemporal and alocational. It has as consequence the temporal-locational proposition:

(b) $\forall t\, \forall l\, (\text{BIRTA}\ (t, l) \rightarrow\ \text{DOG}\ (t, l))$

Whenever and wherever "BIRTA" is a correct description, so too is "DOG".

But (b) doesn't reflect our ordinary use of names in English which supposes that "Birta" refers to something that exists. So for a temporal-locational formalization of the example, it seems we should use:

(c) $\forall t\, \forall l\, [\text{BIRTA}\ (t, l) \rightarrow\ \text{DOG}\ (t, l)] \wedge\ \exists t\, \exists l\, \text{BIRTA}\ (t, l)$

But this could be true in a model if Birta were a human yet we don't have enough locations in the model to distinguish where "BIRTA" is a good description from where "DOG" is a good description. This lack of enough locations or enough times to distinguish Birta-ing from dog-ing is comparable to using the predicate logic criterion of identity to evaluate the equality predicate in classical predicate logic,

which classifies two objects as the same if there are not enough predicates in a model to distinguish them.[3] Only here we are not trying to distinguish things but to distinguish Birta-ing from dog-ing.

We encounter the same problem in trying to formalize a reading of the example as in the present. Perhaps (describing in thing-talk) Birta was previously a beautiful princess. We can restrict (c) to the present rather than all times:

(d) $\forall l$ (BIRTA (now, l) $\rightarrow$ DOG (now, l)) $\wedge$ $\exists l$ (BIRTA (now, l)

But this could be true in ways that we would not accept in thing-talk: Birta could be a human who in the present is always close to a dog, so close that we haven't enough locations in the model to distinguish them.

Perhaps, though, if the example is meant about the present, we could just restrict the subordination relation (a) to hold in the present, using a new form:

(e) (BIRTA sub DOG) (now, here)

This would have as consequence that in any place at any time in the present, if it is correct to assert "BIRTA", then it is correct to assert "DOG", but that need not hold at times outside the present. That is, (d) would be a consequence of (e), but (e) would have primacy: it's not just that Birta-ing happens to be dog-ing now and here, but that the relation is fully conceptual.

But how could subordination change over time? How can a relation of concepts, or a relation on ways of describing the flow of all, or a correct use of words change? Maybe dog-ing was once fish-ing but no longer is fish-ing, now it is mammal-ing? I can imagine how once something was a fish and now is a dog, but I can't understand how all dogs were once fishes, and anyway that's not what's meant. There might not be any dog now that was once a fish. It's that at one time the concept of dog-ing was included in that of fish-ing, but now no longer is. A most basic requirement in our work in both formal and informal logic is that during a discussion, during any particular reasoning, we continue to use words in the same way. Colloquially, words do not change their meanings. Yet to take (e) as true as well as "$\exists t \exists l \neg$ (BIRTA sub DOG) (t, l)" is to say that the relation between the concepts or ways of paying attention in the flow of all given by "BIRTA" and "DOG" have changed, or that what was once not a correct use of words now is a correct use of words.

Perhaps we could formalize the example:

$\forall t \forall l$ ((BIRTA (t, l) $\rightarrow$ (BIRTA + DOG) (t, l))

But this could be true if it just happens that Birta was a human child, born outside in freezing weather, and the moment she was born she was licked and was touched by a dog to keep her warm until she was rescued the next day.

We can formalize the example as an atemporal subordination. We have no way

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[3] See Chapter VI.D of *Predicate Logic.*

to formalize it as a temporal predication. This is not a fault of our logic but a reflection of how different the thing-view of the world is from the mass-process view.

Example 2 Birta is brown.

Analysis This is not meant as an atemporal proposition, unless you think that being brown is an essential property of a thing. Rather, Birta is brown now; later when she gets old she could be mostly grey.

We cannot use:

$$\forall l\,(\text{BIRTA (now, } l) \rightarrow \text{ BROWN (now, } l)) \wedge \exists l\,(\text{BIRTA (now, } l))$$

This could be true in a model if it just happens that in every location now where there is Birta-ing there is also brown gerbil-ing, though she is what we would call in thing-talk black and white.

Instead, consider:

$$\forall l\,[\,\text{BIRTA (now, } l) \rightarrow \text{ (BIRTA + BROWN) (now, } l)\,] \wedge \exists l\,(\text{BIRTA (now, } l))$$

Wherever there is Birta-ing, there also is Birta-ing mixed with brown-ing. Mixing Birta-ing and brown-ing can only be (it seems to me) if Birta is brown, at least if "BIRTA" describes what we would call a single dog. But the example does not assert that Birta is a dog. "Birta" could be used in English as a name for a chipmunk, a duck, a rock, a piston, a herd of buffalo, a collection of cars. Yet for each of those, conceived of in mass-process terms, if mixed with brown-ing would amount to what we would say with "Birta is brown", and the formalization would be good.

Example 3 Birta is brown. Therefore, something is brown.

$$\frac{\forall l\,[\,\text{BIRTA (now, } l) \rightarrow \text{ (BIRTA + BROWN) (now, } l)\,] \wedge \exists l\,(\text{BIRTA (now, } l)}{\exists l\ \text{BROWN (now, } l)}$$

Analysis We can't say that there is something that is brown, but only that there is brown-ing. The example is valid by the principle of subordination of a conjunction to its conjuncts.

Example 4 Birta is not brown.

$$\forall l \neg\,(\text{BIRTA + BROWN) (now, } l) \wedge \exists l\,(\text{BIRTA (now, } l)$$

Analysis If we accept the formalization in Example 2, this follows.

Example 5 Birta is not a cat.

Analysis Perhaps we could use:

$$\neg\,(\text{BIRTA sub CAT})$$

This says that the concept of Birta-ing is not contained in the concept of cat-ing. It does not preclude that (as we would say in thing-talk) Birta is a cat, just as "¬ (BIRTA sub BROWN)" does not preclude that Birta is brown. As some philosophers would say, being a cat is not an essential property of Birta.

Perhaps we should use instead the time-and-location version of subordination:

$\forall t\, \forall l$ (BIRTA $(t, l) \rightarrow$ ¬ (CAT (t, l)))

But that could be false even though Birta is not a cat if there were a field in which Birta is chasing a cat.

Nor can we use:

$\forall t\, \forall l$ ((BIRTA $(t, l) \rightarrow$ ¬ (BIRTA + CAT) (t, l))

Birta might have caught the cat.

There is a solution, however. We could use the negated context version of "BIRTA sub CAT":

¬ [$\forall t\, \forall l$ (BIRTA $(t, l) \rightarrow$ CAT (t, l))]

This is equivalent to:

$\exists t\, \exists l$ (BIRTA $(t, l) \wedge$ ¬ CAT (t, l))

At some time and location there is Birta-ing and not cat-ing, which in our thing-view would amount to Birta not being a cat.

Example 6 Birta is running.

$\exists l$ (BIRTA + RUN) (now, l)

Analysis I think that this is a good formalization. How could there be Birta-ing mixed with running unless, as we conceive it in thing-talk, Birta is running?

If so, then we can similarly formalize:

Birta is eating. $\exists l$ (BIRTA + EAT) (now, l)

Birta is sleeping. $\exists l$ (BIRTA + SLEEP) (now, l)

But, as in Example 1, we cannot formalize:

Birta is (at present) hungry.

Birta is (at present) tired.

There seems to be a big difference between process predicates and classification predicates. In Volume 1, *The Internal Structure of Predicates and Names*, I argued that the difference tracks only how we use those predicates in English without a clear metaphysical basis.[4] Yet we can formalize:

Birta is gentle. $\exists l$ (BIRTA + GENTLE) (now, l)

And "gentle" in English would be a classification predicate, just as is "brown".

[4] See particularly Appendix 3 there.

We derive the concepts for "DOG" and "RUN", for "GENTLE" and "BROWN" from the English words "dog" and "run" and "gentle" and "brown"—we have no other choice in our limited work here. And so we carry with us the baggage of how we conceive of running and being a dog and being gentle and being brown in English into our mass-process language. Whether or how we can formalize a temporal predication or the negation of a temporal predication depends on the meaning of the English words in it that we have imported into our mass-process language.

We have the convention if not illusion that using our thing-language we can focus on just a dog. But we do not have a dog independent of all else even in our conceiving. The dog is brown or black, sitting or standing, wagging her tail or sleeping. Some philosophers say we can focus on the dog simpliciter, that thing independent of its properties. Others say this cannot be: any particular dog cannot be disentangled from its properties, whether those be essential ones or accidental of a particular time. But those properties are not other things. The idea that we can focus our attention on just one thing depends on our distinguishing between things and properties of things. But "DOG", "BROWN", "WAG + TAIL", "BREATHE", "SIT" are all categorematic words, all play the same role in our talk and reasoning about the flow of all. There is no difference between mass, process, quality, or property. There is only the flow of all described in different ways.

In a model of the way the world is now, we do not have contexts in which only "DOG" is true. Even though in English "dog" and "cat" are contrary properties of things (both cannot be true of one thing at a time, but both can be false), both "DOG" and "CAT" can be a good description of a context given by a time and location. There is no description that is complete enough to pick out some of the flow of all that is just dog-ing, even when we restrict our attention to my patio. Nor is it just location: I can point to my patio and say "DOG " intending it to mean during the past week, and you could say "CAT" and be right, for you saw a cat there two days ago. To say that whatever is correctly described with "DOG" is not correctly described with "CAT" is to fall into thing-talk.

We can focus on, pay attention to just dog-ing by using "DOG". That, we indicate, is what we're concerned with in this context, not because it is the only description that can apply in the context, but that is all we are concerned with.

15 Examples of Formalizing

Example 1 Dick (to Tom): Be careful. *The paint there is wet.*

Analysis We can't formalize this in classical predicate logic because "paint" is a mass-term. Here we can use, for appropriate references for the variables:

(a) (PAINT + WET) (*t*, *l*)

This is to take "WET" as a categorematic word: look, see, there "WET". There can be wet-ing, though in English we demand that "wet" describe some stuff: wet paint, wet cat, wet weather.

I think that (a) is preferable to "(PAINT / WET) (*t*, *l*)" because at that time and location "WET" is a good description. And we have that as a consequence of (a) by the principle of subordination and truth in a context and the subordination of a conjunction to its conjuncts. Generally we have[5]:

$$\forall t\, \forall l\, (\, (\mathrm{E} + \mathrm{F})\ (t, l) \rightarrow\ \mathrm{E}\,(t, l)\,)$$

Example 2 Dick (to Tom): *The paint there is drying.*

Analysis It would seem that to formalize this we can use, for appropriate references for the variables:

(a) (PAINT + DRY) (*t*, *l*)

This is correct if we understand "DRY" as a process, dry-ing, which is the reading most compatible with our view of the world as process-mass. But then how can we formalize what Dick said later to Tom, "The paint is dry"?

In English we distinguish between drying and being dry. We might construe "dry" as an adjective to mean the completion of the process of drying and then use the methods of endings and beginnings developed in Volume 2. But there are many times and places where we talk of dry without assuming that the dryness is the end result of some drying. I look at Birta now and say that she's dry without thereby invoking the end of becoming dry. It seems that we need a categorematic word for being dry and not the process of drying and not the end of a process of drying.

We don't have this problem with "DOG", for we don't have in English an idea of becoming dog as opposed to being dog. We don't have this problem with "RUN", for we don't have in English the idea of a run-ing as opposed to run. But we do have this problem with "RED", for we have the word in English "reddening" for becoming red as opposed to being red.

Example 3 Dick and Tom are watching their friend Luis build an adobe home. Luis takes clayey mud, mixes it with straw, forms it into bricks, then dries those bricks in the sun. When he has enough bricks, he builds part of a wall, using more

[5] Compare the principle that + implies $\wedge$ in *The Internal Structure of Predicates and Names.*

of the clayey mud and straw as mortar to bind the bricks together. Tom says to Dick, "I always thought that all mud is bad, but *some mud is useful*.[6]

Analysis How can we say in process-mass logic that some mud is useful?

The word "some" is acting as a quantifier. We talk of some mud, some candy, some water. We also talk of some dogs, some chairs, some houses. In English these kinds of quantification function very differently: one is meant to quantify over "parts" of a mass and the other to quantify over things. Yet they are the same in process-mass logic. We can describe in process-mass logic what "Some mud is useful" is meant to describe:

(a) $\exists t \exists l$ (MUD + USEFUL) (t, l)

We can describe what we would say in English as "Some lions are fierce" with:

(b) $\exists t \exists l$ (LION + FIERCE) (t, l)

The difference between (a) and (b) is not in the nature of the quantification, but in how we conceive and understand "MUD" and "LION" to describe the flux.

Example 4 It's raining.

Analysis We would usually understand the example to mean that it's raining here and now, where it's spoken. Let's suppose we have a name "here" in our semi-formal language.[7] Then we can formalize the example as:

RAIN (now, here)

But what's the point in formalizing this proposition? What relations between it and other propositions need to be respected for us to bother investigating it? From "That's a dog" we can conclude that there is a dog, but what can we conclude from "It's raining"? We can't relate it to anything but "rain": there is no subject.

We might assert:

(RAIN (now, here) $\wedge$ MAN (now, here)) $\rightarrow$ (MAN + WET) (now, here)

We could use this and the formalization of the example together in an inference.

Example 5 Um ser superior cuida de nos.

Analysis I heard a Brazilian friend say this, which we'd roughly translate into English as "A superior being takes care of us". The word "ser" is a verb meaning "to be" in the sense of always or a considerable length of time. Here "ser" is made to do duty as a noun. Yet we cannot use "ser" as a base categorematic word because being, existing is how we describe things. Consider:

DOG (4:43 p.m. April 10 2010, l)

To say this is true of the location of my ranch is to say that there is dog-ing going on at that time and place, and there is nothing more to say about existing. Similarly

[6] Luis suggests that instead of talking they should be helping him.

[7] But see Chapter 44 of *Time and Space in Formal Logic* for problems with using such a name.

for mud-ing and running: there are no assertions of existence but only assertions of categorematic words at times and places.

Example 6 Unicorns don't exist.

Analysis We can become confused trying to reason with this sentence by asking what it is that doesn't exist. We resolve that problem in classical predicate logic by formalizing the example as:

$\neg\, \exists x$ (— is a unicorn) (x)

We use the existence operator to say that there is no thing (in the universe of a model) of which "— is a unicorn" is true. In our process-mass logic we can formalize the example as:

$\neg\, \exists t\, \exists l$ UNICORN (t, l)

There is no time and location (in a model) at which "UNICORN" is a good/accurate/correct description.

Example 7 Leucippus . . . proclaimed that empty space—or the void—exists and that the atoms move in it. This was a revolutionary assertion for his day, since it was tantamount, in the language of the times, to maintaining the existence of that in which nothing exists.

B.A.G. Fuller and Sterling M. McMurrin, *A History of Philosophy*, p. 88

Analysis The oddity here is using the word "exists" for describing the void, as if it were a thing. In the logic of physical things from Volume 2 we don't talk of the void as a thing. Rather, we say:

$\exists t\, \exists l\, \forall x\, \neg$ ((— to exist) ($—_{\text{time}}$, $—_{\text{location}}$, —) (t, l, x))

This is true iff there is a time and place at which no thing—in the universe of the model—exists. We cannot say that there is a time and place at which no thing whatsoever exists because we have no clear idea what we mean by "no thing whatsoever".

In the process-mass view of the world, we don't talk of a void as a thing, nor do we talk of things or masses existing. We could say that there is a time and place at which no base categorematic word applies, but only if we were to allow for quantification over base-mass process words.

Example 8 Snow is white.

SNOW sub WHITE

Analysis The formalization is false (recall Example 1 of Chapter 12, p. 35). Also false is the temporal-locational consequence of it:

$\forall t\, \forall l$ (SNOW (t, l) $\rightarrow$ WHITE (t, l))

If the example is meant, however, as an assertion about the inherent nature of snow, it is an abstraction in the same way a scientific law is an abstraction. It serves

as a guide to us in our understanding of the world: if we ignore all of the stuff we pick up on the sidewalk in New York two days after it snowed except for the crystalline part, if we ignore the soot and dust that is mixed with it, then that stuff is white. The crystalline part is what we take to be "really" snow, in the first steps of scientific analysis.[8] I don't see how to make such assertions based on abstractions in our mass-process logic. But then I don't see how to formulate in predicate logic scientific abstractions. And the example cannot be formalized in predicate logic because it uses the mass term "snow".

Example 9 Suzy: *Swimming is fun.*

Analysis Suzy believes this. But swimming isn't always fun, as Dick knows from when he fell out of a canoe and had to swim in cold water and was exhausted when he arrived at the shore. Similarly we say:

Running is good exercise.

Dieting is difficult.

These are not meant as omnitemporal, omnilocational assertions nor as telling us how to use words. Nor are they meant as talk about abstractions. They are more like rough generalizations: all else being equal, swimming is fun for me, running is good exercise for most people most of the time, dieting is usually difficult for most people. We can't formalize these in our logic.

Example 10 Money is evil.

Analysis When someone says this we don't understand it as a rough generalization but as a comment about the institution of money. The word "money" is shorthand for the social arrangements whereby we exchange goods in our society, and the assertion is a moral one about the nature of those arrangements. Social arrangements are not abstract, for they exist in time and place, but how we could formalize this example I don't know.

Example 11 Suzy had an idea where Puff is.

Analysis Ideas in our common conception in thing-talk are quite odd. It's not clear if we conceive of an idea as being of time or not. Did Einstein's idea of time as relative exist before he had it? Did he discover it or was it part of his brain process? If part of his brain process, how can someone else have the same idea?

In the process view of the world we can take "IDEA" to be a base categorematic word and then say of a time and place:

(SUZY + IDEA) (t, l)

The description of that part of the flux conjoins the descriptions of Suzy-ing and idea-ing. There was idea-ing going on there. Still, we don't have a way to say what an idea might be about. That would require relating categorematic words.

[8] See "Models and Theories" for a fuller discussion of this view of science.

16 Relating

Suzy brought her cat Puff to Dick and Zoe's home. Spot got excited.

(1) Puff is running from Spot.

To formalize this in classical predicate logic, we parse it as an atomic predicate "— ran from —" and two names, "Puff" and "Spot". But that is an inadequate analysis of the structure of (1) because from (1) we can conclude:

Puff is running.

To respect inferences of this sort, we introduced in Volume 1 the idea of a variable restrictor which allows us to parse (1) as a predicate "— is running" and a variable restrictor "from —", with one name "Spot" to fill the blank in that and one name "Puff" to fill the blank in the predicate. Taking account also of time and location (Volume 2), we could formalize (1) as:

$\exists l$ [((— to run $—_{\text{time}}$, $—_{\text{location}}$) / from (Spot)) (Puff, now, l)]

It's wrong to ask how we can formalize (1) in our mass-process logic. That sentence construes experience in terms of things and relations between them. But we can imagine the same scene construed in the view of the world as process. We can treat the names "Puff" and "Spot" as describing Puff-ing and Spot-ing, for which we can use "PUFF" and "SPOT". Dubbing a location for that scene as "DZY" (Dick and Zoe's yard), we have both:

(PUFF + RUN) (now, DZY)

SPOT (now, DZY)

Each of these is a proposition: an accurate description of that time and place in the flow of all. But together they do not describe the scene of (1) because there is no "from". We need to relate these two propositions:

(2) (PUFF + RUN) (now, DZY) <u>from</u> SPOT (now, DZY)

We are not relating a process-mass at one time and place to another process-mass at another time and place, for that is to conceive of process-mass words as names of things. We are relating descriptions of particular times and places in the flow of all. That is, we are relating propositions. The directedness, the relating of "from" is formalized as a propositional connective. Semantically it represents how a part of the flux of all is directed in some way towards another part of the flux of all. But those parts aren't things, they do not stand in a relationship. Indeed, they are not even parts but just some of the flow of all under particular descriptions. They do not have any properties or qualities. But the descriptions can be related with some kind of directionality, directedness, towards-ness, relatedness of the "one" description

to the other, which is as well as I can say it in English. There are two rivers in the Amazon, one clear and one muddy, and as they flow to the ocean they come together. At the juncture you can see the muddy river flowing into the clear river: the joining of the flows is directed in a way we indicate with "into". The processes are not related: they are relating. So, too, Puff-ing + running is relating to Spot-ing, the relating of the flows being described with "from", which I underline to stress it's new role. What we previously viewed as variable restrictors in Volume 1—not only "from" and "into" but also "to", "with", and many others—in a process-mass view we understand as propositional connectives, not logical ones but categorematic.

But (1) is an incomplete description of the scene. In thing-talk, a fuller description is:

(3) Puff is running from Spot barking.

We have no way to formalize this in classical predicate logic even with modifiers: "Spot barking" is not a thing, so no name for that can fill the blank in the variable restrictor "from (—)". Here it is straightforward to give a formal description of the scene:

(PUFF + RUN) (now, DZY) <u>from</u> (SPOT + BARK) (now, DZY)

Or we could use a less complete description which shows that the names are inessential for our use of this categorematic propositional connective:

RUN (now, DZY) <u>from</u> BARK (now, DZY)

A general categorematic propositional connective

Suppose at the time and place of (3),

(4) Dick is watching Spot.

To formalize this in classical predicate logic with modifiers, we use a variable restrictor for direct objects, "obj (—)", and then take account of time and space:

((— to watch $—_{time}$, $—_{location}$) / obj (Spot)) (Dick, now, DZY)

The variable restrictor "obj (—)" is a general kind of directedness we can attach to any predicate.

How can we formalize in our mass-process language what (4) is meant to describe? We don't want to use "obj", for that suggests directedness towards an object. We need a way to indicate an equally general kind of directedness with process-mass. Let's use "<u>directed towards</u>" as a propositional connective. Then we can formalize a process-mass interpretation of the scene that (4) describes:

(5) (DICK + WATCH) (*t*, *l*) <u>directed towards</u> (SPOT) (*t*, *l*)

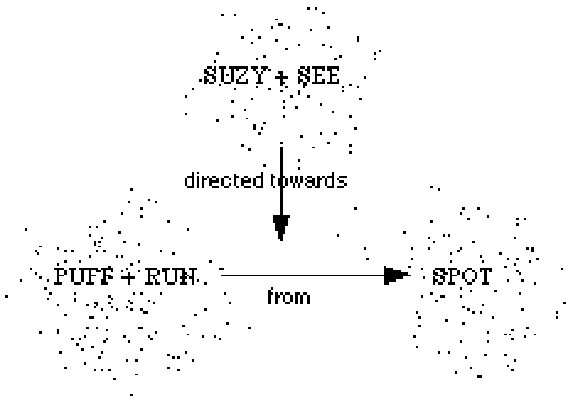
Compounding uses of categorematic propositional connectives
Suppose at the time of (1):

(6) Suzy sees Puff running from Spot.

We have no way to formalize this in classical predicate logic, for what's asserted—in thing-talk—is Suzy seeing what's described by a proposition: (6) is elliptical for "Suzy sees *that* Puff is running from Spot". But in our mass-process logic we can formalize what (6) is meant to describe:

(7) (SUZY + SEE) (now, DZY) directed towards
[(PUFF + RUN) (now, DZY) from (SPOT) (now, DZY)]

To do this we have to allow for a categorematic connective to join an atomic proposition with another that is a categorematic compound. What does that mean? The wff (2) describes a nexus, a view of the flux under distinct descriptions as a unity of some sort. The wff (7) describes a larger nexus. With considerable risk of being misleading, I'll try to picture what (7) describes.



Starting with atomic propositions we can iterate the use of categorematic connectives. This much we should allow. I do not have an idea what it would mean to join with a categorematic connective either quantified wffs or compounds using logical propositional connectives, and to do so would complicate the analyses a great deal. So let's allow no more complexity than iterating categorematic compounds. Moreover, until we find a need to extend our work, let's consider only binary categorematic connectives. I'll set out explicitly how to modify the formal language in the presentation of the entire formal logic in Chapter 22.

Semantics of categorematic propositional connectives
What is the relation of the truth-value of a wff such as (2), (5), or (7) to the semantic properties of its parts? In a model of classical predicate logic we take as primitive which objects "— ran from —" is true of. Here, we will take as primitive which atomic wffs can be connected with "from" to form a true wff.

That we take "applications" of these connectives as primitive is no worse than

taking the application of "(— runs / from —)" as primitive in classical predicate logic with modifiers. It is also no better. If you thought that the presentation of classical predicate logic with modifiers in Volume 1 was lacking because no compositional semantics were given for restrictors modifying predicates, then you'll likely not be satisfied with this presentation in which the truth-value of (2) is not determined by the semantic values of its parts. In both cases I hope that you'll have an incentive to extend this work.

Though we cannot fully explain the truth-value of a proposition of the form "$\mathrm{E}(t, l)$ $\underline{\mathrm{c}}$ $\mathrm{F}(t', l')$" in terms of the semantic values of "$\mathrm{E}(t, l)$" and "$\mathrm{F}(t', l')$" and the categorematic compound $\underline{\mathrm{c}}$, we can note that, as in the examples, if the parts are not true then there is no description, no nexus: we can't relate what isn't. So if the whole is true, the parts must be true. Each categorematic connective c is a kind of conjunction.

Categorematic connectives are conjunctions

$\forall t\, \forall l\, \forall t'\, \forall t'\, [\, (\mathrm{E}(t, l)\ \underline{\mathrm{c}}\ \mathrm{F}(t', t')) \rightarrow (\mathrm{E}(t, l) \wedge \mathrm{F}(t', l'))\,]$

Let's suppose that we understand "BARK" in a way that the following is true.

(8) BARK ≈ DOG + VOCAL

Does it follow that:

(9) ((DICK + THINK) $\underline{\text{of}}$ BARK) ↔ ((DICK + THINK) $\underline{\text{of}}$ (DOG + VOCAL))

How could this fail? Could Dick's thinking be directed only toward bark-ing and not toward dog + vocalizing? Could how we describe the flow of all matter to the truth of a categorematic compound?

If (8) is true, the two categorematic words evoke the same concepts or (8) codifies a correct use of those words. But do they evoke the same concepts for Dick? I don't know how to consider that issue. Some say in thing-talk that Dick could think of Zoe under the name "Zoe" yet not think of her under the descriptive name "the woman that Matilda and Johnny saw in Dick and Zoe's yard". But in (9) there is no what that is the same. There is only the flow of all and different descriptions. Some of those descriptions may be equivalent (for what we're considering). But they aren't of something, of some "what". To say that there is some "what" is to thing-ify parts of the flow of all. Categorematic words are descriptions that apply to some of the flow in some context. Equivalent descriptions are equally useful for describing. We can extend our rule of substitution of equivalent categorematic words to compound wffs.[9]

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[9] It is tempting to say that our logic is then extensional. But the question of extensionality does not arise for there is no thing to be described in two ways. In any case, the term "extensional" is unclear even in predicate logic. See the discussion in Chapter 13 of *The Internal Structure of Predicates and Names.*

17 Examples of Formalizing: Relating

Example 1 Spot was chasing Puff.

$\exists t \exists l$ (((SPOT + CHASE) (t, l) <u>directed towards</u> PUFF (t, l)) $\wedge$ $(t <_{\text{time}}$ now))

Analysis It is easy to misread this and think of "PUFF" as marking a direct object of "(SPOT + CHASE)".

Example 2 Dick wants to eat a steak.

Analysis We cannot formalize this in predicate logic because there is no object towards which Dick's wanting is directed.[10] We cannot formalize this in process-mass logic because there is no process-mass description at a particular time and place towards which Dick's wanting is directed.

Example 3 Dick heard Spot begin to bark.

$\exists t \exists l$ [(DICK + HEAR) (t, l) <u>directed towards</u> (SPOT + BARK) (t, l)
$\wedge$ $(t <_{\text{time}}$ now)) $\wedge$ $\exists t_1$ ($(t_1 <_{\text{time}} t) \wedge$
$\forall t_2$ ($(t_1 <_{\text{time}} t_2 <_{\text{time}} t) \rightarrow \neg \exists l_1$ (SPOT + BARK) (t_2, l_1)]

Analysis We can adapt to process-mass logic the methods of formalizing talk of beginnings and endings we devised for things in time in Volume 2. I'll let you formalize how to say what in thing-talk we might describe with "Dick heard Spot stop barking".

Example 4 Socrates was shorter than Julius Caesar.

Analysis We cannot formalize this in classical predicate logic because we cannot interpret it atemporally: it purports to describe a relation of things in time and space, not some essential attributes of those. But if not atemporal, this is not a proposition because it lacks an indication of what times in the life of Socrates and of Julius Caesar their heights are meant to be compared. Suppose we make that explicit:

(a) Socrates on January 1, 401 BC in Athens was shorter than Julius Caesar on March 1, 44 BC in Rome.

We still have no way to formalize this in the predicate logic of physical things because we have no way to talk of objects at different times and places within a single predication.

Understanding (a) as asserting relation between what we describe as Socrates-ing and as Julius Caesar-ing at those times and places, we can formalize it:

SOCRATES (January 1 401 BC, Athens) <u>shorter than</u>
JULIUS CAESAR (March 1 44 BC, Rome)

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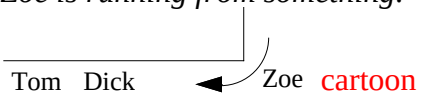
[10] See Chapter 29 of Volume 2.

Here "shorter than" relates two atomic propositions as a categorematic propositional connective to yield a new proposition. What we take in predicate logic to be a comparison, we take here to be a connective of descriptions of parts of the flux of all.

Example 5 Last week Dick and Tom were downtown and suddenly Zoe came running around a corner looking behind her. Dick said to Tom:

(a) *Zoe is running from something.*



Analysis What could that something be? It might be an accident she's seen, a vicious cat, an hallucination, the sparking of a wire, water gushing from a standpipe, skunk odor. Many of the possibilities of what she is running from would not be things in the sense in which we reason about them in predicate logic, so to formalize (a) in predicate logic with a quantifier and variable for "something" would be wrong. How can we formalize (a) if we don't know what Zoe is running from?

The word "something" in (a) should be taken in our mass-process view as some atomic proposition of a time and place. If we were to allow quantification over categorematic words, assuming we have references for the variables, we could use "$\exists$ E ((ZOE + RUN) (t_1, l_1) from E(t_2, l_2))". To do this we would have to allow quantifying over ways to describe the flow of all, which is to reify them: we'd have to have a universe of such ways. That's wrong.

But in the picture at (a) Zoe could be just running from, with a general anxiety of there being a reason to run from without any idea of something she is running from.[11] In (a) the pronoun "something" is as much a dummy as "it" is in "It's raining". Just as informatively Dick could have said:

(b) Zoe is running from.

It's not that (a) and (b) are equivalent: (a) is a bad way we are forced to talk because English is demanding of subjects and objects; (b) is more accurate as a description of the world that Dick is trying to give.

How can we formalize (b) in our mass-process language and logic?[12] We have no guide, for it is not a natural way to talk in English; it arises only by reflection on the nature of what we do and think. I don't know if speakers of some mass-process language conceive of their experience in this way. So all I can do is suggest some possibilities.

We've taken "from" to be a binary propositional connective. To formalize (b) we could use it as a unary connective, too. Then, assuming references for the

[11] See "The Directedness of Emotions" for a fuller discussion.

[12] In classical predicate logic with restrictors perhaps we could allow for a non-variable version of "from", writing "((— run)/from) (Zoe)". That would follow from "((— run)/from (Puff)) (Zoe)", but "$\exists x$ ((— run)/from (x)) (Zoe)" would not follow from it. See p. 73 of Volume 1, *The Internal Structure of Predicates and Names.*

variables, and placing the unary connective before the proposition, we'd have:

(c) $\underline{\text{from}}\ (\text{(ZOE + RUN)}\,(t, l))$

But what sense do we have of (c)? We seem to need some property or quality of the proposition "(ZOE + RUN) (*t*, *l*)" or perhaps of "(ZOE + RUN)". We could take the truth-value of (c) as primitive, assuming as a meaning axiom:

$$\forall t\, \forall l\, \forall t'\, \forall l'\ (\text{E}\,(t, l)\ \underline{\text{from}}\ \text{F}\,(t', l')) \rightarrow\ \underline{\text{from}}\,(\text{E}\,(t, l))$$

Another categorematic propositional connective might have a different meaning axiom.

Alternatively, we can view "from" in (b) as a modifier of "Zoe is running", not modifying the proposition but the categorematic word:

$$(\text{(ZOE + RUN)/from})\ (t_1, l_1)$$

To do this, we'd have to follow up on the idea of using modifiers that aren't categorematic words, which we considered at the end of Chapter 10. We'd have no more problems with this semantically than we had with modifiers in Volume 1 (which was bad enough). But then we'd have two uses of the same word "from":

"$\underline{\text{from}}$" as a binary propositional connective

"from" as a modifier

Each seems well-motivated. They would be connected by:

$$\forall t_1\, \forall l_1\, \forall t_2\, \forall l_2\, [\ (\text{E}(t_1, l_1)\ \underline{\text{from}}\ \text{F}(t_2, l_2))\ \rightarrow\ (\text{E/from})\ (t_1, l_1)\ ]$$

The converse would be false.

Example 6 Spot's barking caused Dick to wake up.

Analysis If the world is made up of things, and propositions are about properties that things have or about relations among things, then it is natural to think that causes are due to things. A cause is or comes from a power in a thing; cause and effect is a relation between things. Since ancient times speakers of our languages have held such a view. How, then, can a view of the world as process allow for cause and effect?

The idea that a cause is a power in a thing, that things have powers inherent in them as if they were creatures with will, has been thoroughly abandoned in the sciences.[13] Requiring that things be at the heart of causal analyses leads people to view waves, weather, indigestion to be things. Or it leads to the view that the things that are causally related are events.

We do not have to take barking to be a thing nor Spot's barking to be an event to give a causal analysis of the example. We can describe the purported cause with:

(a) Spot barked.

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[13] See pp. 41–42 of "Reasoning about Cause and Effect".

We can describe the purported effect with:

(b) Dick woke up.

Whatever causes and effects are, we can describe "them" with propositions.

Those propositions need not be about things. Indeed, it's hard to understand barking as a thing. We can say in our mass-process language what (a) and (b) are meant to describe, where t_1, l_1 and t_2, l_2 are given appropriate references:

(c) (SPOT + BARK) (t_1, l_1)

(d) (DICK + AWAKE) (t_2, l_2)

For there to be cause and effect, (c) must be properly related to (d). That is, "cause" can be understood as a relation on propositions. Can we treat it as a categorematic propositional connective?

(SPOT + BARK) (t_1, l_1) cause (DICK + AWAKE) (t_2, l_2)

For this to be true, we must have[14]:

- The cause happened (the proposition describing it is true).
- The effect happened (the proposition describing it is true).
- The cause precedes the effect.
- It is nearly impossible for the cause to happen (be true) and the effect not to happen (be false)—given some normal conditions.
- The cause makes a difference—if the cause had not happened (been true), the effect would not have happened (been true).
- There is no common cause.

The first two conditions require that cause is a kind of conjunction, like other categorematic connectives. The third requires that $t_1 <_{\text{time}} t_2$. But the fourth and fifth conditions require importing the metalogic into the logic and characterizing normal conditions. That is beyond any obvious formal analysis. So we won't take cause to be a categorematic propositional connective.

Still, this discussion leads us to an understanding of cause-and-effect that is not static, not just a relation of ways the world can be, but a kind of transformation, a kind of function.

Example 7 Functions

Analysis In our ordinary speech, when we talk about a function we understand some kind of process or connection. We say that the volume of a gas is a function of the pressure and temperature, understanding it somehow as a physical or perhaps causal connection. We speak of addition of counting numbers as a function of two

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[14] These conditions are given in my textbook *Critical Thinking*; the analysis is defended in "Reasoning about Cause and Effect".

numbers, understanding that as a procedure to obtain a new number from two given ones. We speak of the tangent of an angle, understanding that as a ratio we calculate from coordinates of the point where the angle intersects a unit circle.[15]

Mathematicians in the 19th century found that the notion of function as process did not help to resolve problems in reasoning with infinitesimals in the calculus. They found that the notion of a function as a procedure impeded deriving theorems about functions. So they abstracted from the notion of a function as process or procedure to view a function as just a correlation between things. The volume of a gas as a quantity is abstracted to be a number that is correlated to numbers giving the pressure and temperature of the gas; measurement and causal relations are ignored. Addition of counting numbers is abstracted to correlating a number to a pair of numbers, for example, 7 is correlated to 3 and 4; actual counting as the basis of addition is ignored. The tangent of an angle is a correlation of a real number to a real number; the procedure of thinking through how the angle intersects a unit circle yielding a ratio of two numbers is ignored. The procedures or processes are what establish the correlations, but once we have the correlations we ignore those procedures or processes. We simply have the correlations and take those to be the functions.

We can recapture the idea of a function as a transformation by considering it as relating how the world is described in one proposition to how the world is described in another proposition. That relating is what we do here with categorematic propositional connectives. Functions treated this way are no different from other categorematic connectives, such as <u>from</u> and <u>to</u>, so long as they do not involve metalogical conditions as "cause" does. The problem of representing partial functions arises in classical predicate logic because functions there are name-makers, and we have to consider, for example, what thing is named with $\sqrt{-2}$ or with $3/0$. Here, we are relating propositions, and what is nonsensical is simply false.

I hope someone will investigate more how we can deal with functions in our mass-process logic of contexts given by times and locations.

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[15] This and the succeeding paragraph are from *The Internal Structure of Predicates and Names.*

18 Examples of Formalizing: Describing Locations

Example 1 Spot is barking to the left of Dick.

Analysis Viewing Spot and Dick as physical things, "to the left of" is a location restrictor: it orients us in space, in this case relative to Dick.[16]

How can we formalize in process-mass logic what the example is meant to describe? There is no Spot and no Dick as things; there is only Spot-ing and Dick-ing at particular times and locations. We need to assert a relation between those.

$\exists l_1 \exists l_2$ ((SPOT + BARK) (now, l_1) <u>to the left of</u> DICK (now, l_2))

Dick-ing here gives us orientation as much as Dick as a thing does in predicate logic.

Example 2 There is a place to the left of Dick where Spot is barking.

Analysis The formalization is the same as for the previous example.

Example 3 The place where Spot is barking is to the left of where Dick is now.

Analysis The difference between this and the previous example is the use of "the". We often talk about "the location" or "the time" as if those were clearly referring when they rarely are. Here, with space and time both understood as process-mass, the idea that we can specify a unique location or a unique time for a wff to be true is more than problematic. There seems to be no good motive and much reason not to add to process-mass logic a method of generating descriptive names for locations and times.

Example 4 The place where Spot bit Puff is to the left of where Dick is now.

$\exists l_1 \exists l_2 \exists t$ (([(SPOT + BITE) (t, l_1) <u>directed towards</u> PUFF (t, l_1)]
<u>to the left of</u> DICK (now, l_2)) $\wedge$ ($t <_{\text{time}}$ now))

Analysis The formalization says that there is a time before now and a place where Spot-ing and bite-ing was directed towards Puff-ing and that place is to the left of where Dick-ing is going on now.

In taking this formalization we would not be talking about the location in relation to another location so much as the location where something "happened" in relation to where something else is now "happening". This is to take <u>to the left of</u> as a relation between descriptions of the flux and not as between locations. But how else could we do this if we are to have orientation?

Example 5 Spot is barking in front of Dick and between Suzy and Tom.
Therefore, there is a place that is in front of Dick and between Suzy and Tom.

Analysis How shall we take account of "between"? If we extend process-mass

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[16] See Chapter 45 of Volume 2, *Time and Space in Formal Logic*.

logic to allow for "between" to be treated as a ternary categorematic propositional connective, we could formalize the premise as:

$$\exists l_1 \exists l_2 \exists l_3 \exists l_4 [((\text{SPOT} + \text{BARK}) (\text{now}, l_1) \ \underline{\text{in front of}} \ \text{DICK} (\text{now}, l_2))$$
$$\wedge ((\text{SPOT} + \text{BARK}) (\text{now}, l_1) \ \underline{\text{between}} \ (\text{SUZY} (\text{now}, l_3) , \text{TOM} (\text{now}, l_4))]$$

Or, since evaluating "between" does not depend on orientation, we could adopt a pure location-predicate "between ($—_{\text{location}}$, $—_{\text{location}}$, $—_{\text{location}}$)". But that is problematic, as we saw in Example 9 of Chapter 46 of *Time and Space in Formal Logic*.

How shall we formalize the conclusion? It says that there is a place, and describes it not by "what is happening" there but in relation to other places which are given by mass-process descriptions. The only way we can talk about a location absent a mass-process description that holds of it is to use W_{location}, and that is not helpful here.

We could introduce a location-existence predicate "exist ($—_{\text{location}}$)" into process-mass logic as part of the pure location vocabulary. Then we could allow categorematic propositional connectives to connect not only categorematic atomic wffs but also atomic propositions of the form "exist ($—_{\text{location}}$) (l)". In that case we could formalize the conclusion of the example as:

$$\exists l_1 \exists l_2 \exists l_3 \exists l_4 (((\text{exist} (—_{\text{location}}) (l_1) \ \underline{\text{in front of}} \ \text{DICK} (p, l_2))$$
$$\wedge ((\text{exist} (—_{\text{location}}) (l_1) \ \underline{\text{between}} \ (\text{SUZY} (p, l_3) , \text{TOM} (p, l_4)))$$

Or we could allow for locational connectives that would relate a term for a location to an atomic wff. Then we could formalize the conclusion as:

$$\exists l_1 \exists l_2 \exists l_3 \exists l_4 ((l_1 \ \underline{\text{in front of}} \ \text{DICK} (p, l_2)) \wedge$$
$$(l_1 \ \underline{\text{between}} \ (\text{SUZY} (p, l_3), \text{TOM} (p, l_4)))$$

Either approach would complicate our process-mass logic considerably.

Or we could say that our talk of times and places as things is meant only to facilitate reasoning about the flow of all. We need to be able to make assertions about the existence of times and places solely in relation to other times and places in order to ensure that our conception of time and space is adequate for reasoning about the flux of all. Those assumptions are made in the pure language of space and time. It is not a flaw in our theory that we cannot relate a location absent any description of what is happening there to other locations that are picked out by what is happening at them. In that case, we can do no more than use the formalization of the premise as the formalization of the conclusion, too.

19 Talking of Stability and Change

We can talk of dog-ing here and now, we can talk of dog-ing there and then. But we have no way to talk of the continuity of this dog-ing with that dog-ing, nor how this dog-ing was conjoined with sleep-ing and now is conjoined with awake-ing. Can we do so without falling into thing-talk?

When I point and say "this mud" I direct your attention to some place now where that description applies. I am pointing to the mud-ing, not to some part of the flow of all; the time and place only facilitate how to pay attention. When I talk about the barking near the corral last night, I direct your attention to some place at some time in the past where "BARK" applies. I am not pointing to a part of the flow of all but to where and when "BARK" applies.

If "DOG(*t, l*)" is true at a time in a place, then that wff directs our attention to "the" dog-ing that is going on there, whether that be, in thing-talk, one dog barking, or two dogs sleeping, or a pack of bloodhounds following a scent baying, or a dead dog. Likewise, if "MUD(*t, l*)" is true at a time in a place, then that wff directs our attention to "the" mud-ing there, whether that be a single spot of mud on a sidewalk, or a rain-drenched clayey path in a forest, or several patches of mud in a field. If "RUN(*t, l*)" is true at a time and place, then that wff directs our attention to "the" running there, whether that be what we would describe in thing-talk as Birta running after a rabbit or 302 people running in a marathon.

So to say "this dog-ing", "this mud-ing", "this bark-ing", "this running" need not involve a conception of thing(s), except to the extent that we speak of times and locations as if they were things in order to direct attention.

Local categorematic words

I can direct your attention to the dog-ing now here in the patio with:

(1) DOG (—$_{time}$, —$_{location}$) (now, patio)

That's true. I'd like to tell you that this dog-ing was also in the corral last night. How can I do that?

We can use (1) to create a new categorematic word:

(2) ⟦ this DOG (—$_{time}$, —$_{location}$) (now, patio) ⟧

This establishes a concept, a way of talking of the flow of all, as much as "RUN". And that's because (1) is true. Normally we do not inquire into how a concept of a categorematic word is established or given. Simply, "DOG" can be used by us as a way of describing in the flow of all. But with (2), the new concept is established with a time and location relative to our understanding of "DOG". It is a local version of "DOG", a *local categorematic word*. Abbreviating, we can write (2) as:

(3) ⟦ this DOG (now, patio) ⟧

Certainly (3) is true of now and the patio, for that's exactly how it was meant to describe. That is, the following is true:

⟦ this DOG (now, patio) ⟧ (now, patio)

The word (3) gives a restricted concept of dog-ing. That is dog-ing, too. We have either conceptually or as a correct use of words:

(4) ⟦ this DOG (now, patio) ⟧ sub DOG

Now I can say that the dog-ing in the patio now was in the corral last night:

(5) ⟦ this DOG (now, patio) ⟧ (last night, corral)

The categorematic concept word established in relation to now and the patio with "DOG" is true of last night in the patio.

Suppose (5) is true. That does not mean that (3) describes "all" of the dog-ing in the corral last night. It could be that only Birta is in the patio now, while last night both she and Bidu were in the corral. What we have by (4) is that the following is true:

DOG (last night, corral)

So we can form:

⟦ this DOG (last night, corral) ⟧

And this is true of last night in the corral:

⟦ this DOG (last night, corral) ⟧ (last night, corral)

But as just noted, the following can be false:

⟦ this DOG (last night, corral) ⟧ (now, patio)

Change

Now we can say:

(⟦ this DOG (now, patio) ⟧ + AWAKE) (now, patio)

∧ (⟦ this DOG (now, patio) ⟧ + SLEEP) (last night, corral)

Suppose this is true. That doesn't mean that all of the dog-ing last night in the corral was mixed with sleep-ing, for Bidu could have been awake and alert for coyotes. Nor does it mean that Birta has one property now and had a different property last night. It means that the dog-ing here now was mixed with awake-ing, and the same dog-ing was mixed with sleep-ing last night in the corral. We pick out what we shall consider stable in our talk with a local categorematic word, and then find out whether it applies, how it applies, and with what it applies at various times in various locations. That (5) is true does not mean that the dog-ing now in the patio existed last night in the corral. That is thing-talk. We're not talking about existence. A local version of a categorematic word does not pick out something that exists. It only provides a way to describe with a time-and-place restricted categorematic word.

To make that clearer, suppose I direct your attention to an odd smell now in the patio with:

SMELL (now, patio)

Then I can say that the same smell was in the corral last night:

⟦ this SMELL (now, patio) ⟧ (last night, corral)

Suppose then that the following is true:

(⟦ this SMELL (now, patio) ⟧ + WIND) (now, patio)

∧ (⟦ this SMELL (now, patio) ⟧ + WATER) (last night, corral)

That does not mean that a thing that's now in the patio had one property and that in the corral last night it had another, unless you think that any time we use "this", as in "this smell", we have to be talking about a thing with properties.

Open local categorematic words

We can use variables for times and locations to form local categorematic words, too, so long as the variables are assigned reference. For example, let:

(6) t_1 have reference now
l_1 have reference the patio
t_2 have reference last night
l_2 have reference the area in the corral

According to what we assumed above, the following are then true:

⟦this DOG (t_1, l_1)⟧

⟦this DOG (t_2, l_2)⟧

These are "open" local categorematic formulas. Each establishes a concept, a categorematic word for describing, but only because the variables in them are assigned reference as at (6). According to what we assumed above, the following are true:

⟦this DOG (t_1, l_1)⟧ (t_1, l_1)

⟦this DOG (t_1, l_1)⟧ (t_2, l_2)

⟦this DOG (t_2, l_2)⟧ (t_2, l_2)

But we need not have that the following is true:

⟦this DOG (t_2, l_2)⟧ (t_1, l_1)

To say that relative to the references at (6), "⟦this DOG (t_1, l_1)⟧" establishes the same concept as "⟦ this DOG (now, patio) ⟧", that they are equivalent under subordination, we'll have to talk about subordination relative to an assignment of references, which we'll do in the next chapter.

The same?

But what do we mean by "the same"? What do we mean by saying that the same smell-ing that's in the patio now was in the corral last night? The same mud-ing? The same running? What is the same dog-ing?

What we mean by "the same" is primitive, fundamental, whether talking of things or the flow of all. In thing-talk we become tongue-tied trying to say what we mean by identity, by this thing here being the same as that thing to which we referred earlier. We talk about properties of "the thing", and essential properties, but at best what we can do is try to draw out some important characteristics of that notion, not reduce it to any other semantic one. Here, we can talk about continuity over time, about our perception of stability in the flux, but at best what we can do is try to draw out some important characteristics of our idea of sameness of the flow under particular descriptions.

Whatever this notion of sameness is, it is not an identity of things, even if what we are talking about in our process language would be characterized as a thing in English. No two dog-ings are identical. There is no idea here of unchanging stability, for all is flux and process. Rather, we mean that describing this in the flux (pointing) is somehow continuous with describing that in the flux (pointing). There is some likeness that justifies our saying that the descriptions pick out the same.

Suppose "DOG" is true at time t and place p because the only dog-ing going on there is Birta-ing; in thing-talk it's true because Birta is there and she's the only dog there. Suppose it is also true at t′, p′ for the same reason. We think in thing-talk that Birta is a stable thing and that she is located at the one time and place and at the other, so that what this wff picks out at the first time is a slice of Birta in time, and what it picks out at the second is another slice of Birta in time. But how can those be the same if, for example, t is very much longer than t′ or if at the first time Birta is barking and at the second she is sleeping? It is not time-slices of a thing that are the same. It is what we are paying attention to, dog-ing only, that is the same. What we have is the flux of all under various descriptions, and the process, the dog-ing at the one time and place is the same process, dog-ing, at the other time and place—though that way makes it sound as if we were considering processes as things. What we have are words we use for descriptions, and sometimes a local version of one can be understood to apply in another context. This sameness, this likeness does not assume a notion of a thing continuous in time.

Continuity of description in time

With *t* standing for now as I write and *l* for the location of my patio, each of the following is true:

DOG (*t, l*)

MUD (*t, l*)

SLEEP (*t, l*)

So relative to those references for *t* and *l*, each of the following local categorematic words can be used for describing:

(7) (a) ⟦this DOG (*t*, *l*)⟧

(b) ⟦this MUD (*t*, *l*)⟧

(c) ⟦this SLEEP (*t*, *l*)⟧

What (7a) describes at that time and place is the entirety of dog-ing there; what (7b) describes at that time and place is the entirety of mud-ing there; what (7c) describes is the entirety of sleeping there.

The entirety of dog-ing has continuity in time: if, in thing-talk, one of the dogs were to be killed by a mountain lion, then at that later time (7a) is not true in any place. If all of the patches of mud in the patio dry up and then it rains and is muddy in exactly the same places again, at the later time it's not the same mud-ing as before: (7b) at that later time is not true in any place. If Birta is sleeping in the patio, wakes up, then goes back to sleep again, it's not the same sleeping as before: (7c) at that later time is not true in any place..

The notion of "same" in the world as process requires continuity in time. If any of the describing words at (7) is true of a later time, it must be true at all times in between the time we assigned to *t* and the later time—in some place or places.

But what place or places? Suppose that "⟦this DOG (*t*, *l*)⟧" is true now in Dick and Zoe's yard and was true earlier in the day in a small location around the bus stop (in thing-talk, Spot is here now and was at the bus stop earlier). It seems there should be a continuous path of locations from the bus stop to Dick and Zoe's yard in which "⟦this DOG (*t*, *l*)⟧" is true in a continuous path of time from that earlier time to now. I don't know how to formulate such a condition with the vocabulary of space and time we have, just as I failed to see how to enforce a condition of continuity in location for objects in the classical predicate logic physical things in Chapter 36 of Volume 2.

Complex local categorematic words

Dick asks Zoe why she's screwing up her nose. She says she smells the same skunk odor that was here yesterday. Suppose:

t_3 has reference when Dick and Zoe are talking

l_3 has reference a small location where Dick and Zoe are

t_4 has reference the day previous to when Dick and Zoe are talking

With those references, we can use mass-process talk for what Zoe described in thing-talk:

(ZOE + SMELL) (t_3, l_3) <u>directed towards</u>

⟦this (ODOR / SKUNK) (t_4, l_3)⟧ (t_3, l_3)

That is, Zoe-ing + smelling was directed toward skunk odor-ing that was the same as the skunk odor-ing at the earlier time. From this, because categorematic connectives are conjunctions, we can conclude:

⟦ this (ODOR / SKUNK) (t_4, l_3) ⟧ (t_3, l_3)

And from this we can conclude both:

(ODOR / SKUNK) (t_3, l_3)

(ODOR / SKUNK) (t_4, l_3)

We can also use a local categorematic word as a modifier, as in:

ODOR / ⟦ this SKUNK (t_4, l_3) ⟧

And we can use that in a wff:

(ODOR / ⟦ this SKUNK (t_4, l_3) ⟧) (w, p).

In thing-talk we might say the odor at w, p is from the skunk (or skunks) at t_4, l_3.

We can use local categorematic words in the same way as we used any categorematic word.

Mistakes

We can be mistaken in our describing. You might think that some sheep are in the corral now and use:

(8) ⟦ this SHEEP (now, corral) ⟧

But I've moved them to the pasture. So (8) describes nothing. So it can't be true of any time or place. That is, for any references for t and l, the following is false:

⟦ this SHEEP (now, corral) ⟧ (t, l)

This seems like "UNICORN". But there's a difference. "UNICORN" is meaningful, just not a good description of any time and place. But (8) does not establish a concept we can use to see if it's true or false of a time and place. It's nonsense—like talking about the man who wasn't there on the stair yesterday. As nonsense it yields nonsense when used as part of any other categorematic word. So the following are nonsense:

DOG + ⟦ this SHEEP (now, corral) ⟧

IDEA / ⟦ this SHEEP (now, corral) ⟧

It makes no sense to talk of dog-ing mixed with some sheep-ing when we're not describing any sheep-ing. We can't talk about an idea of the sheep-ing in the corral if we're mistaken in describing sheep-ing. What is nonsense is false, as we've assumed in all our work. So for any assignment of references to t and l, the following are false:

(DOG + ⟦ this SHEEP (now, corral) ⟧) (*t*, *l*)

(IDEA / ⟦ this SHEEP (now, corral) ⟧) (*t*, *l*)

In contrast, "IDEA / UNICORN" can be true of a time and place, say, yesterday when Suzy was at home thinking of unicorns.

Why not get rid of such nonsense by requiring that E (*t*, *l*) has to be true in order for "⟦ this E (*t*, *l*) ⟧" to be a part of our language? If *t* or *l* is a variable, then for some assignments E (*t*, *l*) could be true and so "⟦ this E (*t*, *l*) ⟧" would be meaningful, while for some assignments E (*t*, *l*) could be false and it would not be meaningful. Giving semantic criteria for an inscription to be a formula leads to major confusions: we can't give an inductive definition of the formal language.[17]

Some definitions and principles

Here are definitions and statements of principles culled from the discussions above; these will be used in extending our formal logic in Chapter 22.

Local categorematic words Given any categorematic word E, we can form the expression:

⟦ this E ($—_{time}$, $—_{location}$) (*t*, *l*)⟧

It is a *local categorematic word* or *this-formula*. Informally, we abbreviate it as ⟦ this E (*t*, *l*)⟧.

If either of *t* or *l* is a variable, then ⟦ this E (*t*, *l*)⟧ is an *open local categorematic word* or *open this-formula.* Given an assignment of references σ, ⟦ this E (*t*, *l*)⟧ is a *local categorematic word relative to* σ.

What we previously called categorematic words are distinguished as those in which no term appears; we call those *ordinary* categorematic words.

Non-describing local categorematic words

For any assignment of references σ, if E (*t*, *l*) is false, then ⟦ this E (*t*, *l*)⟧ is *non-describing relative to* σ.

If neither *t* nor *l* is a variable, then ⟦ this E (*t*, *l*)⟧ is simply *non-describing*.

Non-describing local categorematic words are nil

If ⟦ this E (*t*, *l*)⟧ is non-describing relative to σ, then for any atomic proposition A in which ⟦ this E (*t*, *l*)⟧ appears, and for any τ that agrees with σ on all the variables that appear in E (*t*, *l*), A is false relative to τ.

Note that from this, if ⟦ this E (*t*, *l*)⟧ (*w*, *p*) is true relative to σ, so is E (*t*, *l*).

Describing local categorematic words

For any assignment of references σ, if E (*t*, *l*) is true, then ⟦ this E (*t*, *l*)⟧ is *describing relative to* σ.

If neither *t* nor *l* is a variable, then ⟦ this E (*t*, *l*)⟧ is *describing*.

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[17] See "On the Error in Frege's Proof that Names Denote".

Describing categorematic words are true of their time and location
If E (*t*, *l*) is true relative to σ, then:

a. ⟦ this E (*t*, *l*) ⟧ (*t*, *l*) is true relative to σ.

b. For any τ that agrees with σ on all the variables that appear in E (*t*, *l*), if ⟦ this E (*t*, *l*) ⟧ (*w*, *p*) is true relative to τ, then so is E(*w*, *p*).

Continuity of descriptions in time
If relative to σ, ⟦ this E (*t*, *l*) ⟧ (t_1, l_1) is true, and ⟦ this E (*t*, *l*) ⟧ (t_2, l_2) is true, and $t_1 <_{\text{time}} t_3 <_{\text{time}} t_2$ is true relative to σ, then there is some τ that agrees with σ on all of *t*, *l*, t_1, l_1, t_2, l_2, and t_3 such that ⟦ this E (*t*, *l*) ⟧ (t_3, l_3) is true.

Aside: Quantifiers in this-*words?*
Why not form this-words from descriptions that contain quantifiers? For example,

⟦ this (∃ l_1 (HUSBAND(now, l_1) of WOMAN(now, *l*))) ⟧

This describes what we in English would call a married woman—or perhaps many married women. I think. Actually, I get confused about how to understand this expression. And if we allow quantifiers to appear in a this-word, we will involve ourselves in considerable technical complications. So let's not pursue that here.

Aside: Reference
It might seem that this chapter is about referring, and what I have introduced are referring and non-referring categorematic words which obey many of the same principles as referring and non-referring names in Volume 1, *The Internal Structure of Predicates and Names*.
In my essay "Nouns and Verbs" I explain why I avoid the terms "reference" and "referring" in favor of "description" and "describing" in discussions of mass-process languages.

20 Local Categorematic Words and Subordination

Subordination relative to an assignment of references
The principle of subordination and truth in a context for ordinary mass-process words is:

(E sub F) → ∀*t* ∀*l* (E (*t*, *l*) → F (*t*, *l*))

Now we have local categorematic words that contain variables. So subordinations can be relative to an assignment of references. For example, the following is true or false only relative to an assignment of references to *w* and *p*:

(1) ⟦ this SHEEP (*w*, *p*) ⟧ sub ANIMAL

If "⟦ this SHEEP (*w*, *p*) ⟧" is non-describing, that is, if "SHEEP (*w*, *p*)" is false, then (1) is false, for non-describing words are nil. If "⟦ this SHEEP (*w*, *p*) ⟧" is describing, then (1) could be true or false. If true, it follows that relative to any τ that agrees with σ on *w* and *p*: if ⟦ this SHEEP (*w*, *p*) ⟧ (*t*, *l*), then ANIMAL (*t*, *l*).
To extend the principle of subordination and truth in a context, we need only add quantification to take account of variables.

Subordination and truth in a context

∀ . . . [(E sub F) → ∀*t* ∀*l* (E (*t*, *l*) → F (*t*, *l*))]

We needn't put in a clause that in case E is a local categorematic word then it is describing, for if it is not then the antecedent of the consequent is false.

Subordination of local words and the words on which they are based
Suppose that "⟦ this SHEEP (*w*, *p*) ⟧" is describing relative to a particular assignment of references. Then that word establishes a restricted concept of sheep-ing. That is sheep-ing too. We have either conceptually or as a correct use of words:

⟦ this SHEEP (*w*, *p*) ⟧ sub SHEEP

We also have:

(2) SHEEP sub ANIMAL

So it follows by transitivity that:

⟦ this SHEEP (*w*, *p*) ⟧ sub ANIMAL

But more, if "SHEEP (*w*, *p*)" is true, then by (2), "ANIMAL (*w*, *p*)" is true. And we have:

⟦ this SHEEP (*w*, *p*) ⟧ sub ⟦ this ANIMAL (*w*, *p*) ⟧

The sheep-ing at *w*, *p* is not only subordinate to animal-ing, it is subordinate to this local version of animal-ing

To state the general principles, we have to note that these relations hold only if the local categorematic words are describing.

Subordination of a local categorematic word to the categorematic word on which it is based

$\forall \ldots$ (E $(t, l) \rightarrow$ (⟦ this E (t, l) ⟧ sub E))

Subordination yields subordination of the local categorematic words

$\forall \ldots$ [(E sub F) $\wedge$ E $(t, l) \rightarrow$ (⟦ this E (t, l) ⟧ sub ⟦ this F (t, l) ⟧)]

Note that the last principle does not hold if a time or location term is not the same in the two local categorematic words. For example, the following is true:

DOG sub DOG

But if (in thing-talk) in the patio now there is just Birta, and last night only Bidu was in the corral, the following is false:

⟦ this DOG (last night, corral) ⟧ sub ⟦ this DOG (now, patio) ⟧

The concept of Bidu-ing is not included in the concept of Birta-ing.

Identity and equivalence of categorematic words
Suppose that the following are both true:

(3) ⟦ this DOG (now, patio) ⟧ (last night, corral)

⟦ this DOG (last night, corral) ⟧ (now, patio)

Then. since "⟦ this DOG (now, patio) ⟧" describes the entirety of dog-ing now in the patio, and "⟦ this DOG (last night, corral) ⟧" describes the entirety of the dog-ing last night in the corral, those two words must establish the same concept and be suitable to be used one in place of the other. That is:

(4) ⟦ this DOG (now, patio) ⟧ $\approx$ ⟦ this DOG (last night, corral) ⟧

But how can these be conceptually the same if they involve different times and places? The times and places direct us to how to find the conception; they are not part of the conception. Compare:

(5) DOG $\approx$ CANINE / DOMESTIC

The words direct us to the conception, but the words are not part of the conception; we need not be complete nominalists. We might not agree on what we mean by "a concept" or "conception" or "conceiving" or "correct use of words". But we have enough in common in our use of those phrases, and perhaps even in how we understand them, to use them together in our reasoning and try to come to some clearer idea of what we can agree on by codifying our correct uses.

From (4) by the relation of subordination to truth in context, for any assignment of references to *t* and *l*, we have:

⟦ this DOG (now, patio) ⟧ (*t*, *l*) ↔ ⟦ this DOG (last night, corral) ⟧ (*t*, *l*)

But remember that the context version of subordination is a consequence of an equivalence; it does not imply an equivalence. For example, the following might be true in a model:

∀*t* ∀*l* (⟦ this DOG (last night, corral) ⟧ (*t*, *l*) ↔
⟦ this SHEEP (last night, corral) ⟧ (*t*, *l*))

We might not have enough times and locations in the model to distinguish that dog-ing from that sheep-ing (think: the sheep dog and the sheep are always together). But that does not mean that "⟦ this DOG (last night, corral) ⟧" is equivalent to "⟦ this SHEEP (last night, corral) ⟧". The conceptions are different, not established solely by the times and locations but by the times and locations relative to different categorematic words.

We have (4) because of (3). But equally, by (4) and (5) and the principle that subordination yields subordination of the local categorematic words, we have:

⟦ this DOG (now, patio) ⟧ ≈ ⟦ this (CANINE / DOMESTIC) (last night, corral) ⟧

Here is the general principle.

Identity and equivalence

∀ . . . [((E ≈ F) ∧ ⟦ this E (*t*, *l*) ⟧ (*w*, *p*) ∧ ⟦ this F (*w*, *p*) ⟧ (*t*, *l*))
↔ (⟦ this E (*t*, *l*) ⟧) ≈ (⟦ this F (*w*, *p*) ⟧)]

We've assumed in our logic that the use of time and location terms is extensional: same reference, same semantic values result. So if *t* is assigned reference last night, and *l* is assigned reference the area in the corral, if "DOG (last night, corral)" is true, then:

⟦ this DOG (last night, corral) ⟧ ≈ ⟦ this DOG (*t*, *l*) ⟧

Iteration of the this-*operator*

To return to one of our earlier examples, suppose that the following is referring because now just Birta is in the patio:

⟦ this DOG (now, patio) ⟧

Now consider:

⟦ this DOG (now, patio) ⟧ (last night, corral)

For this to be true, there had to be Birta-ing last night in the corral. But there could be more dog-ing. Perhaps Bidu was in the corral last night, too. So consider:

⟦ this ⟦ this DOG (now, patio) ⟧ (last night, corral) ⟧

This establishes the same concept as "⟦ this DOG (now, patio) ⟧". That is,

⟦ this ⟦ this DOG (now, patio) ⟧ (last night, corral) ⟧ ≈ ⟦ this DOG (now, patio) ⟧

For any ordinary categorematic words,

(7) $\forall t\, \forall l\, \forall w\, \forall p$ [E$(t, l) \wedge$ ⟦ this E(t, l) ⟧ $(w, p) \rightarrow$
⟦ this ⟦ this E(t, l) ⟧ (w, p) ⟧ ≈ ⟦ this E(t, l) ⟧]

However, as in the example, the following can fail:

E$(t, l) \wedge$ ⟦ this E(t, l) ⟧ $(w, p) \rightarrow$
⟦ this ⟦ this E(t, l) ⟧ (w, p) ⟧ ≈ ⟦ this E(w, p) ⟧

This would hold relative to an assignment of references only if ⟦ this E (t, l)⟧ ≈ ⟦ this E (w, p)⟧ .

What if E is or contains a local categorematic word? For example, suppose in (7), E is "⟦ this SHEEP (last night, corral) ⟧":

$\forall t, l$ [(⟦ this SHEEP (last night, corral) ⟧ $(t, l) \wedge$
⟦ this ⟦ this SHEEP (last night, corral) ⟧ (t, l) ⟧ (w, p)) →
⟦ this ⟦ this ⟦ this SHEEP (last night, corral) ⟧ (t, l) ⟧ (w, p) ⟧ ≈
⟦ this ⟦ this SHEEP (last night, corral) ⟧ (t, l) ⟧]

This is true: the reasoning is the same, just harder to parse.

In (7), for "⟦ this E(t, l) ⟧ (w, p)" to be true, both "⟦ this E(t, l) ⟧" must be true, and hence "E(t, l)" is true, and any local categorematic word that appears in E must be describing. So for the general principle we can take the following.

Iterations of local categorematic words

$\forall$ . . . (⟦ this E(t, l) ⟧ $(w, p) \rightarrow$
⟦ this ⟦ this E(t, l) ⟧ (w, p) ⟧ ≈ ⟦ this E(t, l) ⟧])

Disjoint locations

We can have:

⟦ this DOG (last night, corral) ⟧ ≈ ⟦ this DOG (now, patio) ⟧

The dog-ing in the corral last night and the dog-ing in the patio now are the same. But since the patio and the corral are completely separate, we can't have:

⟦ this DOG (now, corral) ⟧ ≈ ⟦ this DOG (now, patio) ⟧

What's dog-ing in the one location can't be the same as the dog-ing in the other separate location. That's not just for dog-ing. Suppose that there is a lake with two

coves called "Shady Cove" and "Sunny Cove". Then even though "WATER" is true in both those locations now, the following is false:

$$[\![\text{this WATER (now, Shady Cove)}]\!] \approx [\![\text{this WATER (now, Sunny Cove)}]\!]$$

The first picks out "the" water in Shady Cove and the second picks out "the" water in Sunny Cove.

If two local categorematic words are equivalent at the same time, then there must be some overlap in their locations. And it must be in some such overlap that the describing is established.

Disjoint locations at the same time yield different descriptions

$\forall \ldots [\, [\![\text{this E } (t, l_1)]\!] \approx [\![\text{this E } (t, l_2)]\!] \rightarrow$

$(\exists p \, (W_{\text{location}}(p, l_1) \wedge W_{\text{location}}(p, l_2)) \wedge [\![\text{this E } (t, p)]\!] \approx [\![\text{this E } (t, l_1)]\!]) \,]$

Other subordination principles

All the previous principles we adopted for subordination for ordinary categorematic words apply to ones that are or contain local categorematic words so long as the latter are referring and we take the universal closure.

Reflexivity If E is an ordinary categorematic word, (E sub E).

$\forall \ldots (\text{E } (t, l) \rightarrow [\![\text{this E } (t, l)]\!] \text{ sub } [\![\text{this E } (t, l)]\!])$

I don't see how to combine these two principles into one because "UNICORN sub UNICORN" is true, yet "UNICORN (t, l)" is not true for any assignment of references.

Note, then, that "E sub E" is true iff every local categorematic word appearing in E is describing.

Subordination is transitive

$\forall \ldots [((E_1 \text{ sub } E_2) \wedge (E_2 \text{ sub } E_3)) \rightarrow (E_1 \text{ sub } E_3)]$

Commutativity of conjuncts in a conjunction

$\forall \ldots (\text{A (F)} \leftrightarrow \text{A (F}' \text{)})$

where F is a conjunction of categorematic words; E_1 and E_2 are conjuncts in F; F′ is F with E_1 and E_2 replacing each other; A(F) is an atomic proposition in which F appears; and A(F′) is A(F) with some but not necessarily all occurrences of F replaced with F′.

Subordination of a conjunction to its conjuncts

If E is a conjunction of categorematic words, and F is E with one of its conjuncts deleted, then $\forall \ldots (\text{E sub E} \rightarrow (\text{E sub F}))$.

Conjunction preserves subordination

$\forall \ldots \ ((E_1 \text{ sub } E_2) \wedge (F \text{ sub } F)) \rightarrow ((F + E_1) \text{ sub } (F + E_2))$

Subordination of a modified categorematic word

$\forall \ldots \ ((E/F) \text{ sub } (E/F)) \ \rightarrow \ (E/F) \text{ sub } E$

Modifying a categorematic word by itself adds nothing

$\forall \ldots (E \text{ sub } E \rightarrow \ (E/E \approx E)$

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21 Names

Descriptive names?

In Chapter 5 we agreed that we can use a name from our ordinary thing-talk, such as Zoe, as a categorematic word. To assert "ZOE" about here and now is to say here and now Zoe-ing. When "DZY" stands for Dick and Zoe's yard, we can assert:

ZOE (now, DZY)

SPOT (now, DZY)

There, now, the flux viewed locally as Zoe-ing; there, now, the flux viewed locally as Spot-ing.

Can we use a description of Zoe and abbreviate that as a name rather than using "ZOE" as a categorematic word? Right now in Dick and Zoe's yard, there is woman-ing: "WOMAN(now, DZY)" is true. From our thing-talk perspective that's because Zoe is there and no other woman is there. Matilda and her boyfriend Johnny are just outside the gate. Before they go in, Matilda says to Johnny, "See that woman there, that's Zoe." "Oh, I see," says Johnny, "now I know who Zoe is." So it seems that whenever he or Matilda use the name "ZOE", Johnny can understand it as:

(1) ⟦this WOMAN(now, DZY)⟧

But that's not how Shondel, a friend of Zoe's mother, understands "ZOE". She learned that name when she and Zoe's mother met at a park 15 years ago when Zoe was 8 years old. Let *t* stand for that time and *l* stand for a location of that park where the only female child at that time is Zoe. Then Shondel associates "ZOE" with:

(2) ⟦this GIRL(*t*, *l*)⟧

But (1) and (2) can't be equivalent, because at *t* and *l* there is no woman-ing, and now in Dick and Zoe's yard there is no girl-ing.

Things, as we conceive of them in English, can have different properties at different times. But there is no thing in mass-process talk that can have those different properties. There are only the descriptions, nothing to "hang them on". We cannot have descriptive names.

Characterizing names

Consider:

⟦this CAT(t_1, l_1)⟧

⟦this CAT(t_2, l_2)⟧

Suppose that relative to an assignment of references both are describing. It does not follow that relative to that assignment of references:

⟦this CAT(t_1, l_1)⟧ sub ⟦this CAT(t_2, l_2)⟧

It might be that at t_1 in l_1 there were two feral cats fighting and at t_2 in l_2 there was only Puff sleeping.

In contrast, suppose that relative to some assignment of references τ, both of the following are referring:

$[\![\text{this BIRTA } (t_1, l_1)]\!]$

$[\![\text{this BIRTA } (t_2, l_2)]\!]$

Need the following be true relative to τ?

$[\![\text{this BIRTA } (t_1, l_1)]\!] \text{ sub } [\![\text{this BIRTA } (t_2, l_2)]\!])$

What if $\tau(t_1)$ is much shorter than $\tau(t_2)$? What if $\tau(l_1)$ is completely disjoint from $\tau(l_2)$? That doesn't matter. We are describing the same: in both places there is Birta-ing. It's alway the same Birta-ing.

For any τ, if BIRTA (t, l) is true relative to τ, then so is,
$[\![\text{this BIRTA } (t, l)]\!] \approx \text{BIRTA}$.

If N is a categorematic word that is meant as a name, it should satisfy the following.

Unity of names $\forall t \, \forall l \, (\text{N}(t, l) \rightarrow [\![\text{this N}(t, l)]\!] \approx \text{N})$

For example, "PEGASUS" satisfies this condition, since it is true of no time and location. It is not vacuous, like "$[\![\text{this CAT (now, corral)}]\!]$", which is non-describing, for "PEGASUS sub HORSE" is true, either in terms of concepts or according to our correct use of these words. Also, "DRAGON" satisfies this condition of unity of names. It, too, is not vacuous since "DRAGON sub BREATHE" is true. We use and understand "PEGASUS" as a name, but we don't use or understand "DRAGON" as a name: there is nothing in the concept that precludes "DRAGON" being true of disjoint places at the same time. It just happens that there are no dragons. To classify a word as a name, the condition of the unity of names has to be true not just in a model but across all models. We can't enforce that with any axiom. So instead we'll use separate symbols for names and then require that in any model the condition of unity of names applies for those categorematic words..

The principle of continuity of descriptions for local categorematic words says that if a local categorematic word is true at some time and again at some later time, then it is true at all times between those. We want that to apply to names, too,.

Continuity in time for names

$\forall \ldots \, (\text{N}(t, l) \wedge \text{N}(t', l') \wedge (t <_{\text{time}} w <_{\text{time}} t')) \rightarrow \exists p \, (\text{N}(w, p))]$

With local mass-process words we have the condition that disjoint locations at the same time yield different descriptions. Here we can't have different descriptions using "ZOE". Rather, "ZOE" can't apply in disjoint locations at the same time: it

can't be Zoe-ing in Dick and Zoe's yard and at the bus stop down the road at the noon. And if the locations are not disjoint, then the Zoe-ing is a good description of the common part.

A name cannot be a good description in disjoint locations at the same time

$\forall t \, \forall \, l \, \forall \, l' \, [\, (\mathrm{N}(t, l) \wedge \mathrm{N}(t, l')) \rightarrow (\exists p \, \mathrm{W}(p, l) \wedge \mathrm{W}(p, l') \wedge \mathrm{N}(t, p)) \,]$

Now we are ready to formulate a logic of mass-process with contexts given by times and locations.

22 A Formal Logic of Mass-Process with Contexts Given by Times and Locations

The formal language

Vocabulary

base categorematic word symbols

ordinary $B_1, B_2, \ldots, B_n, \ldots$

name $N_1, N_2, \ldots, N_n, \ldots$

categorematic word conjoiner $+$

describing word maker this

subordination symbol sub

binary categorematic propositional connective symbols $\underline{c}_1, \underline{c}_2, \ldots$

logical propositional connectives $\neg, \rightarrow, \wedge, \vee$

time name symbols $b_0, b_1, \ldots$
time variables $t_0, t_1, \ldots$
} *time terms*

time order predicate $\text{—} <_{\text{time}} \text{—}$

time part predicate $W_{\text{time}}(\text{—}, \text{—})$

time equality predicate $\equiv_{\text{time}}$

location name symbols $e_0, e_1, \ldots$
location variables $l_0, l_1, \ldots$
} *location terms*

location part predicate $W_{\text{location}}(\text{—}, \text{—})$

location equality predicate $\equiv_{\text{location}}$

quantifiers $\forall, \exists$

Punctuation

parentheses ( )

time-marked blank —_{time}

location-marked blank $\text{—}_{\text{location}}$

slash /

comma ,

double brackets ⟦ ⟧

We adopt the following meta-variables:

B, B´ stand for base categorematic word symbols, ordinary or name.

N, N´ stand for name symbols.

E, E´, E_1, E_1 . . . , F, F´ stand for formal categorematic words as defined below.

t, *t´*, *w*, *w´*, w_1, w_2, . . . stand for time terms.

l, *l´*, *p*, *p´*, p_0, p_1, p_2, . . . stand for location terms.

c, c´ stand for categorematic propositional connective symbols.

Categorematic words

i. If B is a base categorematic word symbol, then (B) is a formal categorematic word of degree 1.

ii. If $E_1, \ldots, E_r$ are formal categorematic words where $r \geq 2$ and n is the maximum of the degrees of $E_1, \ldots, E_r$, then $(E_1 + \ldots + E_r)$ is a formal categorematic word of degree $n + 1$. It is a *conjunction* of formal categorematic words. Each of $E_1, \ldots, E_r$ is a *conjunct* of it.

iii. If E and F are formal categorematic words, and the maximum of the degrees of E and F is *n*, then E / F is a formal categorematic word of degree $n + 1$. It is a *modified* formal categorematic word. F is the *modifier* and E is the word *modified.*

iv. If E is a formal categorematic word, then the following is a *local formal categorematic word* of degree $n + 1$:

⟦ this E $(—_{\text{time}}, —_{\text{location}})$ (t, l) ⟧

It is also called a *this-word.*

v. A concatenation of symbols is a formal categorematic word iff for some *n* it is a formal categorematic word of degree *n*.

A formal categorematic word is *open* if it contains a variable; otherwise, it is *closed*. If E is a formal categorematic word that is not of degree 1, then E is a *complex* formal categorematic word.

I'll let you show that there is one and only one way to parse each formal categorematic word.

Well-formed formulas (*wffs*)

i. If E and F are formal categorematic words, then (E sub F) is a wff of length 1.

ii. If E is a formal categorematic word, *t* a time term, and *l* a location term, then (E $(—_{\text{time}}, —_{\text{location}})$ (t, l)) is a wff of length 1.

iii. If t and w are time terms, each of the following is a wff of length 1:

$((— <_{\text{time}} —)\,(t, w))$

$(W_{\text{time}}(—, —)\,(t, w))$

$((— \equiv_{\text{time}} —)\,(t, w))$

iv. If l and p are location terms, each of the following is a wff of length 1:

$(W_{\text{location}}(—, —)\,(l, p))$

$((— \equiv_{\text{location}} —)\,(l, p))$

Each occurrence of a variable in a wff of length 1 is *free*.

v. If $\underline{c}$ is a propositional connective symbol, and E and F are formal mass-process words, then the following is a wff of length 2:

$(E\,(—_{\text{time}}, —_{\text{location}})\,(t, l))\ \underline{c}\ (F\,(—_{\text{time}}, —_{\text{location}})\,(w, p))$

It is a *categorematic* compound. Each occurrence of a variable in it is free.

vi. If A and B are categorematic compounds , and the maximum of the lengths of A and B is n, and $\underline{c}$ is a propositional connective symbol, then $(A\ \underline{c}\ B)$ is a wff of length $n + 1$. It is a *categorematic compound*. Each occurrence of a variable in it is free.

v. If A is a wff of length n, then $(\neg A)$ is a wff of length $n + 1$. An occurrence of a variable in $(\neg A)$ is free iff it is free in A.

vi. If A and B are wffs, and the maximum of the lengths of A and B is n, then each of $(A \rightarrow B)$ and $(A \wedge B)$ and $(A \vee B)$ is a wff of length $n + 1$. An occurrence of a variable in $(A \rightarrow B)$ is free iff the corresponding occurrence of the variable in A or in B is free, and similarly for $(A \wedge B)$ and $(A \vee B)$.

vii. If A is a wff of length n and some occurrence of a time variable t is free in A, then each of $(\forall t\ A)$ and $(\exists t\ A)$ is a wff of length $n + 1$. An occurrence of a variable in either $(\forall t\ A)$ or $(\exists t\ A)$ is free iff the variable is not t and the corresponding occurrence in A is free.

viii. If A is a wff of length n and some occurrence of a location variable l is free in A, then each of $(\forall l\ A)$ and $(\exists l\ A)$ is a wff of length $n + 1$. An occurrence of a variable in either $(\forall l\ A)$ or $(\exists l\ A)$ is free iff the variable is not l and the corresponding occurrence in A is free.

ix. A concatenation of symbols is a *wff* iff it is a wff of length n for some $n \geq 1$.

I'll let you prove the unique readability of wffs.

A wff of length 1 is *atomic*; all other wffs are *compound.* A wff is *closed* if there is no occurrence of a variable free in it; otherwise it is *open*.

In $(\forall t\,\mathrm{A})$ the initial $\forall t$ has *scope* A and *binds* each free occurrence of t in A, and that occurrence is *bound* by that quantifier; similarly for $(\exists t\,\mathrm{A})$.

In $(\forall l\,\mathrm{A})$ the initial $\forall l$ has *scope* A and *binds* each free occurrence of l in A, and that occurrence is *bound* by that quantifier; similarly for $(\exists l\,\mathrm{A})$.

The universal closure of a wff

Let $t_{m_1}, \ldots, t_{m_s}$ be a list of all the time variables that occur free in A such that $m_1 < \cdots < m_s$. Let $l_{i_1}, \ldots, l_{i_v}$ be a list of all the location variables that occur free in A such that $i_1 < \cdots < i_v$. The *universal closure* of A is:

$$\forall \ldots \mathrm{A} \equiv_{\mathrm{Def}} \forall t_{m_1} \cdots \forall t_{m_s}\ \forall l_{i_1} \cdots \forall l_{i_v}\,\mathrm{A}$$

We take the usual definitions of substituting for variables from classical predicate logic. We define:

$$\mathrm{E} \approx \mathrm{F} \equiv_{\mathrm{Def}} (\,(\mathrm{E\ sub\ F}) \wedge (\mathrm{F\ sub\ E})\,)$$

We adopt the usual conventions on informally deleting parentheses from propositional logic and the usual informal abbreviations from predicate logic and from the pure logic of space and time. We can informally delete parentheses around a base categorematic word when that isn't likely to lead to confusion. Informally, we abbreviate:

$$[\![\,\mathrm{this\ E}(\__{\mathrm{time}}, \__{\mathrm{location}})\,(t, l)\,]\!] \text{ as } [\![\,\mathrm{this\ E}(t, l)]\!].$$

$$(\mathrm{E}(\__{\mathrm{time}}, \__{\mathrm{location}})\,(t, l)) \text{ as } \mathrm{E}\,(t, l).$$

Realizations and semi-formal languages

A categorematic word of our "ordinary" language, or a time name, or a location name, or a categorematic connective is *simple* iff it contains no proper part that we could formalize as a categorematic word, a time name, a location name, a logical propositional connective, a categorematic propositional connective, a quantifier, or some combination of those with +, /, or the this-operator.

A *realization* of the formal language is an assignment of:

- Simple categorematic words to some or all of the ordinary categorematic symbols.
- Simple names to none, some, or all of the name categorematic symbols.
- Simple time names to none, some, or all of the time symbols.
- Simple location names to none, some, or all of the location symbols.
- Simple categorematic connectives to none, some, or all of categorematic propositional connective symbols.

No word can be assigned to more than one symbol.

The definition of *realization of a formal wff , semi-formal wff,* and *semi-formal language* are the usual ones.

Models

Given a realization, we define a model.

Universes

We adopt two universes:

A universe T which is a non-empty collection of things that are *times*.

A universe P which is a non-empty collection of things that are *locations*.

Nothing is in both T and P.

We use the following meta-variables:

$t, t', t_1, t_2, \ldots, w, w', w_1, w_2, \ldots$ stand for elements of T.

$p, p', p_1, p_2, \ldots$ stand for elements of P.

Relations on the universes

There is a binary relation $<$ on T.

There is a binary relation W_{time} on T.

There is a binary relation W_{location} on P.

$X_<(w_1, w_2) \equiv_{\text{Def}}$ there is some t such that $W_{\text{time}}(t, w_1)$ and $W_{\text{time}}(t, w_2)$, and there is some t' such that $W_{\text{time}}(t', w_1)$ and for all w such that $W_{\text{time}}(w, w_2)$, $t' < w$.

These relations satisfy the following conditions.

W_{time} *is a part-whole relation*

$W_{\text{time}}(t, t)$

If $W_{\text{time}}(t, w)$ and $W_{\text{time}}(w, t)$, then $t = w$.

If $W_{\text{time}}(t_1, t_2)$ and $W_{\text{time}}(t_2, t_3)$ then $W_{\text{time}}(t_1, t_3)$.

Parts determine times

[For all w, $W_{\text{time}}(w, t)$ iff $W_{\text{time}}(w, t')$] iff $t = t'$.

$<$ *is an ordering*

Not $t < t$.

If $t_1 < w$ and $w < t_2$, then $t_1 < t_2$.

Parts and wholes are unrelated in the ordering

If $W_{\text{time}}(t_1, t_2)$ then neither $t_1 < t_2$ nor $t_2 < t_1$.

Parts of times are related to other times in the ordering as the whole is related

If $W_{time}(t_1, t_2)$ and $t_2 < t_3$ then $t_1 < t_3$.

If $W_{time}(t_1, t_2)$ and $t_3 < t_2$ then $t_3 < t_1$.

Times are intervals (*sequentially connected*)

If $W_{time}(t_1, w)$ and $W_{time}(t_2, w)$ then: $W_{time}(t_1, t_2)$ or $W_{time}(t_2, t_1)$ or $X_<(t_1, t_2)$ or $X_<(t_2, t_1)$ or $t_1 < t_2$ or $t_2 < t_1$.

If $W_{time}(t_1, w)$ and $W_{time}(t_2, w)$ and $t_1 < t_2$, then for every t_3 such that $t_1 < t_3 < t_2$, $W_{time}(t_3, w)$.

Overlapping times are not related in the ordering

If $X_<(t, w)$, then neither $w_1 < w_2$ nor $w_2 < w_1$.

$W_{location}$ *is a part-whole relation*

$W_{location}(p, p)$

If $W_{location}(p, p')$ and $W_{location}(p', p)$, then $p = p'$.

If $W_{location}(p_1, p_2)$ and $W_{location}(p_2, p_3)$ then $W_{location}(p_1, p_3)$.

Parts determine locations

[For all p , $W_{time}(p, p_1)$ iff $W_{time}(p, p_2)$] iff $p_1 = p_2$.

Assignments of references

An assignment of references σ assigns:

To each time variable *t* a time σ(*t*) from T.

To each time name b a time σ(b) from T such that for every assignment of references τ, σ(b) = τ(b).

To each location variable *l* a location σ(*l*) from P.

To each location name e a location σ(e) from P such that for every assignment of references τ, σ(e) = τ(e).

Completeness of the collection of assignments of references

There is at least one assignment of references.

For every assignment of references σ, and every time variable *t*, and every time in T, either σ assigns that time to *t* or there is an assignment τ that differs from σ only in that it assigns that time to *t*.

For every assignment of references σ, and every location variable *l*, and every location in P, either σ assigns that location to *l* or there is an assignment τ that differs from σ only in that it assigns that location to *l*.

We say that τ *agrees with* σ on *t* if τ(*t*) = σ(*t*), and in that case we write $\tau \sim_t \sigma$, and similarly for more variables and for location variables.

Satisfaction of atomic wffs
For each assignment of references σ, there is a valuation υ_σ that assigns to each atomic wff A a truth-value T or F. If true, we write $\upsilon_\sigma(A) = T$ or $\upsilon_\sigma \models A$; if false we write $\upsilon_\sigma(A) = F$ or $\upsilon_\sigma \not\models A$. If A is $E(\_\__{time}, \_\__{location})\ (t, l)$ and $\upsilon_\sigma(A) = T$, we say that $E(\_\__{time}, \_\__{location})\ (t, l)$ is *true at time* σ(*t*) *in location* σ(*l*).

I will write $\upsilon_\sigma \models E \approx F$ as shorthand for $\upsilon_\sigma \models E$ sub F and $\upsilon_\sigma \models F$ sub E.

The valuations of the atomic wffs satisfy the following conditions.

$\upsilon_\sigma \models (w <_{time} t)$ iff σ(*w*) < σ(*t*).

$\upsilon_\sigma \models W_{time}(w, t)$ iff $W_{time}(\sigma(w), \sigma(t))$.

$\upsilon_\sigma \models W_{location}(l, p)$ iff $W_{location}(\sigma(l), \sigma(p))$.

$\upsilon_\sigma \models w \equiv_{time} t$ iff σ(*w*) is the same time as σ(*t*).

$\upsilon_\sigma \models l \equiv_{location} p$ iff σ(*l*) is the same location as σ(*p*).

The extensionality condition
Let *w* be a time term in A (if any), and *p* a location term in A (if any). For any σ and τ that agree on all the variables in A except possibly *w* and *p*, if σ(*w*) = τ(*w´*) and σ(*p*) = τ(*p´*), then

$\upsilon_\sigma \models A$ iff $\upsilon_\tau \models A(w´/w, p´/p)$.

Outward closure of truth for locations
If $\upsilon_\sigma \models E(t, l)$, then for all τ that agree with σ on all the variables in $E(t, l)$, if $\upsilon_\tau \models W_{location}(l, p)$, then $\upsilon_\tau \models E(t, p/l)$.

Downward and upward closure of truth in time at a location
$\upsilon_\sigma \models E(t, l)$ iff for all τ that agree with σ on all the variables in $E(t, l)$ except possibly *t*, if $\upsilon_\tau \models W_{time}(w , t)$, then $\upsilon_\tau \models E(w/t, l)$.

Subordination and truth in a context
If $\upsilon_\sigma \models$ (E sub F) then for every assignment of references τ that agrees with σ on all the variables in E and F: if $\upsilon_\tau \models (E(\_\__{time}, \_\__{location})\ (t, l))$, then $\upsilon_\tau \models (F(\_\__{time}, \_\__{location})\ (t, l))$.

Substitution of equivalent categorematic words
If $\upsilon_\sigma \models E \approx F$, and A(E) is an atomic proposition in which E appears, and A(F) is A(E) with F replacing some but not necessarily all occurrences of E, then $\upsilon_\sigma A(F)$ iff $\upsilon_\sigma A(E)$.

Non-describing local categorematic words are nil
If $\upsilon_\sigma \not\models E(t, l)$, and A is an atomic proposition in which ⟦this $E(t, l)$⟧ appears, then for any τ that agrees with σ on all the variables that appear in $E(t, l)$, $\upsilon_\tau \not\models A$.

Describing local categorematic words are true of their time and location
If $\upsilon_\sigma \vDash \mathrm{E}(t, l)$, then:

a. $\upsilon_\sigma \vDash [\![\text{this E } (t, l)]\!] \ (t, l)$.

b. For any τ that agrees with σ on all the variables that appear in $\mathrm{E}(t, l)$, if $\upsilon_\tau \vDash [\![\text{this E } (t, l)]\!] \ (w, p)$, then $\upsilon_\tau \vDash \mathrm{E}(w, p)$.

Continuity of description in time (*including names*)
If E is a formal local categorematic word or a name symbol, and $\upsilon_\sigma \vDash \mathrm{E}(w_1, p_1)$ and $\upsilon_\sigma \vDash \mathrm{E}(w_2, p_2)$, and $\sigma(w_1) < \sigma(w_3) < \sigma(w_2)$, then there is some τ that agrees with σ on w_3 such that $\upsilon_\tau \vDash \mathrm{E}(w_3, p_3)$.

Subordination of a local categorematic word to the categorematic word on which it is based
If $\upsilon_\sigma \vDash \mathrm{E}(t, l)$, then $\upsilon_\sigma ([\![\text{this E } (t, l)]\!] \text{ sub E})$.

Subordination yields subordination of the local categorematic words
If $\upsilon_\sigma \vDash \mathrm{E} \text{ sub F}$ and $\upsilon_\sigma \vDash \mathrm{E}(t, l)$, then
$\upsilon_\sigma \vDash [\![\text{this E } (t, l)]\!] \text{ sub } [\![\text{this F } (t, l)]\!]$.

Identity and equivalence
If $\upsilon_\sigma \vDash \mathrm{E} \approx \mathrm{F}$, and $\upsilon_\sigma \vDash [\![\text{this E}(t, l)]\!] \ (w, p)$,
and $\upsilon_\sigma \vDash [\![\text{this F}(w, p)]\!] \ (t, l)$, then
$\upsilon_\sigma \vDash [\![\text{this E}(t, l)]\!] \approx [\![\text{this F}(w, p)]\!]$.

Iterations of local categorematic words
If $\upsilon_\sigma \vDash [\![\underline{\text{this}} \ \mathrm{E}(t, l)]\!] \ (w, p)$, then
$\upsilon_\sigma \vDash [\![\text{this } [\![\text{this E } (t, l)]\!] \ (w, p)]\!] \approx [\![\text{this E } (t, l)]\!]$.

Disjoint locations at the same time yield different descriptions
If $\upsilon_\sigma \vDash [\![\text{this E } (t, l_1)]\!] \approx [\![\text{this E } (t, l_2)]\!]$, then there is some τ that agrees with σ on all the variables in E as well as t, l_1 and l_2 such that $\upsilon_\sigma \vDash (\mathrm{W}_{\text{location}}(p, l_1)$, and $\upsilon_\sigma \vDash \mathrm{W}_{\text{location}}(p, l_2))$, and $\upsilon_\sigma \vDash [\![\text{this E } (t, p)]\!] \approx [\![\text{this E } (t, l_1)]\!])$.

A name cannot be true at some time in disjoint locations
If $\upsilon_\sigma \vDash \mathrm{N}(t, l_1)$ and $\upsilon_\sigma \vDash \mathrm{N}(t, l_2)$, then there is some τ that agrees with σ on t, l_1 and l_2 such that $\upsilon_\sigma \vDash (\mathrm{W}_{\text{location}}(l_3, l_1)$, and $\upsilon_\sigma \vDash \mathrm{W}_{\text{location}}(l_3, l_2))$, and $\mathrm{N}(t, l_3)$.

Subordination is reflexive
a. If E is an ordinary categorematic word, then $\upsilon_\sigma \vDash (\mathrm{E} \text{ sub E})$.
b. If $\upsilon_\sigma \vDash \mathrm{E}(t, l)$, then $\upsilon_\sigma \vDash [\![\underline{\text{this}} \ \mathrm{E}(t, l)]\!] \text{ sub } [\![\underline{\text{this}} \ \mathrm{E}(t, l)]\!]$.

Subordination is transitive
If $\upsilon_\sigma \vDash (E_1 \text{ sub } E_2)$ and $\upsilon_\sigma \vDash (E_2 \text{ sub } E_3)$, then $\upsilon_\sigma \vDash (E_1 \text{ sub } E_3)$.

Commutativity of conjuncts in a conjunction
$\upsilon_\sigma \vDash A(F)$ iff $\upsilon_\sigma \vDash A(F')$

where F is a conjunction of categorematic words; E_1 and E_2 are conjuncts in F; and F′ is F with E_1 and E_2 replacing each other; and A(F) is an atomic proposition in which F appears; and A(F′) is A(F) with some but not necessarily all occurrences of F replaced with F′.

Subordination of a conjunction to its conjuncts
If E is a conjunction of categorematic words, and F is E with one or more of its conjuncts deleted, and $\upsilon_\sigma \vDash (E \text{ sub } E)$, then $\upsilon_\sigma \vDash (E \text{ sub } F)$.

Conjunction preserves subordination
If $\upsilon_\sigma \vDash (E_1 \text{ sub } E_2)$ and $\upsilon_\sigma \vDash (F \text{ sub } F)$, then $\upsilon_\sigma \vDash (F + E_1) \text{ sub } (F + E_2)$.

Subordination of a modified categorematic word
If $\upsilon_\sigma \vDash (E / F) \text{ sub } (E / F)$, then $\upsilon_\sigma \vDash ((E / F) \text{ sub } E)$.

Modifying a categorematic word by itself adds nothing
If $\upsilon_\sigma \vDash (E \text{ sub } E)$, then $\upsilon_\sigma \vDash (E / E \approx E)$.

Unity of names
If $\upsilon_\sigma \vDash N(t, l)$, then $\upsilon_\tau \vDash [\![\underline{\text{this}}\ N(t, l)]\!] \approx N$.

Satisfaction of compound wffs
The valuations for all assignments of wffs are extended to all wffs simultaneously in the usual way for classical predicate logic:

$\upsilon_\sigma(\neg A) = \mathsf{T}$ iff $\upsilon_\sigma(A) = \mathsf{F}$

$\upsilon_\sigma(A \rightarrow B) = \mathsf{T}$ iff $\upsilon_\sigma(A) = \mathsf{F}$ or $\upsilon_\sigma(B) = \mathsf{T}$

$\upsilon_\sigma(A \wedge B) = \mathsf{T}$ iff $\upsilon_\sigma(A) = \mathsf{T}$ and $\upsilon_\sigma(B) = \mathsf{T}$

$\upsilon_\sigma(A \wedge B) = \mathsf{T}$ iff $\upsilon_\sigma(A) = \mathsf{T}$ or $\upsilon_\sigma(B) = \mathsf{T}$

$\upsilon_\sigma(\exists t\ A) = \mathsf{T}$ iff for some τ such that $\tau \sim_t \sigma$, $\upsilon_\tau(A) = \mathsf{T}$

$\upsilon_\sigma(\forall t\ A) = \mathsf{T}$ iff for every τ such that $\tau \sim_t \sigma$, $\upsilon_\tau(A) = \mathsf{T}$

$\upsilon_\sigma(\exists l\ A) = \mathsf{T}$ iff for some τ such that $\tau \sim_l \sigma$, $\upsilon_\tau(A) = \mathsf{T}$

$\upsilon_\sigma(\forall l\ A) = \mathsf{T}$ iff for every τ such that $\tau \sim_l \sigma$, $\upsilon_\tau(A) = \mathsf{T}$

Categorematic connectives are conjunctions
If $\upsilon_\sigma \vDash (E(t, l) \subseteq F(t', l'))$, then $\upsilon_\sigma \vDash E(t, l)$ and $\upsilon_\sigma \vDash F(t', l')$.

We write $\upsilon_\sigma \models A$ to mean that $\upsilon_\sigma(A) = \mathsf{T}$, and $\upsilon_\sigma \not\models A$ to mean $\upsilon_\sigma(A) = \mathsf{F}$.

The *valuation* υ is defined on all closed wffs A by:

$\upsilon(A) = \mathsf{T}$ iff every σ, $\upsilon_\sigma(A) = \mathsf{T}$.

A *model* M is a realization, universe of times, universe of locations, complete collection of assignments of references, valuations for atomic wffs satisfying the conditions above, extension of the valuations to all wffs by the inductive definition, and the valuation on all closed wffs. A proposition A of the semi-formal language is *true in the model* iff $\upsilon(A) = \mathsf{T}$, in which case we write $\mathsf{M} \models A$. Otherwise, A is *false in the model*, and we write $\mathsf{M} \not\models A$.

A formal wff A is *valid* or a *tautology* iff in every model its realization is true; in that case we write $\models A$. The formal inference Γ therefore A is *valid*, written $\Gamma \models A$, means that there is no model in which the realizations of all the wffs in Γ are true and the realization of A is false. These definitions are extended to semi-formal wffs via formal wffs of which they are realizations.

The formal logic of mass-process with contexts given by times and locations
The formal language, definition of models, definition of tautology, and definition of semantic consequence constitute the logic **IXNAHFROBITZ**.

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23 An Axiom System

A, B, C stand for any wffs of the formal language L of the last chapter.

Propositional axioms
The axiom schemes of classical propositional logic**:**

$\forall \ldots (\neg A \rightarrow (A \rightarrow B))$

$\forall \ldots (B \rightarrow (A \rightarrow B))$

$\forall \ldots ((A \rightarrow B) \rightarrow ((\neg A \rightarrow B) \rightarrow B))$

$\forall \ldots ((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)))$

$\forall \ldots (A \rightarrow (B \rightarrow (A \wedge B)))$

$\forall \ldots ((A \wedge B) \rightarrow A)$

$\forall \ldots ((A \wedge B) \rightarrow B)$

$\forall \ldots (A \rightarrow (A \vee B))$

$\forall \ldots (B \rightarrow (A \vee B))$

$\forall \ldots ((A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow ((A \vee B) \rightarrow C)))$

Axioms governing ∀

1. a. $\forall \ldots (\forall t\,(A \rightarrow B) \rightarrow (\forall t\,A \rightarrow \forall t\,B))$
   if t is free in both A and B
   b. $\forall \ldots (\forall t\,(A \rightarrow B) \rightarrow (\forall t\,A \rightarrow B))$
   if t is free in A and not free in B
   c. $\forall \ldots (\forall t\,(A \rightarrow B) \rightarrow (A \rightarrow \forall t\,B))$
   if t is free in B and not free in A
2. $\forall \ldots (\forall t\,\forall w\,A \rightarrow \forall w\,\forall t\,A)$
3. $\forall \ldots (\forall t\,A(t) \rightarrow A(w/t))$
   if w is free for t in A
4. a. $\forall \ldots (\forall l\,(A \rightarrow B) \rightarrow (\forall l\,A \rightarrow \forall l\,B))$
   if l is free in both A and B
   b. $\forall \ldots (\forall l\,(A \rightarrow B) \rightarrow (\forall l\,A \rightarrow B))$
   if l is free in A and not free in B
   c. $\forall \ldots (\forall l\,(A \rightarrow B) \rightarrow (A \rightarrow \forall l\,B))$
   if l is free in B and not free in A
5. $\forall \ldots (\forall l\,\forall p\,A \rightarrow \forall p\,\forall l\,A)$

6. $\forall \ldots (\forall l\, \mathrm{A}(l) \rightarrow \mathrm{A}(p/l))$
   where p is free for l in A
7. $\forall \ldots (\forall t\, \forall l\, \mathrm{A} \rightarrow \forall l\, \forall t\, \mathrm{A})$
8. $\forall \ldots (\forall l\, \forall t\, \mathrm{A} \rightarrow \forall t\, \forall l\, \mathrm{A})$

Axioms for the relation between ∀ and ∃

9. a. $\forall \ldots (\exists t\, \mathrm{A} \rightarrow \neg \forall t \neg \mathrm{A})$
   b. $\forall \ldots (\neg \forall t \neg \mathrm{A} \rightarrow \exists t\, \mathrm{A})$
10. a. $\forall \ldots (\exists l\, \mathrm{A} \rightarrow \neg \forall l \neg \mathrm{A})$
    b. $\forall \ldots (\neg \forall l \neg \mathrm{A} \rightarrow \exists l\, \mathrm{A})$

Axioms for equality and extensionality

11. $\forall t\, (t \equiv_{\text{time}} t)$
12. $\forall \ldots \forall t\, \forall w\, (t \equiv_{\text{time}} w \rightarrow (\mathrm{A}(t) \rightarrow \mathrm{A}(w/t)))$
    where A is atomic and w replaces some
    but not necessarily all occurrences of t in A
13. $\forall l\, (l \equiv_{\text{location}} l)$
14. $\forall \ldots \forall l\, \forall p\, (l \equiv_{\text{location}} p \rightarrow (\mathrm{A}(l) \rightarrow \mathrm{A}(p/l)))$
    where A is atomic and p replaces some
    but not necessarily all occurrences of l in A

Axioms for time

W_{time} *is a part-whole relation*

$\forall t_1\, \mathrm{W}_{\text{time}}(t_1, t_1)$

$\forall t_1\, \forall t_2\, (\mathrm{W}_{\text{time}}(t_1, t_2) \wedge \mathrm{W}_{\text{time}}(t_2, t_1) \rightarrow (t_1 \equiv_{\text{time}} t_2))$

$\forall t_1\, \forall t_2\, \forall t_3\, (\mathrm{W}_{\text{time}}(t_1, t_2) \wedge \mathrm{W}_{\text{time}}(t_2, t_3) \rightarrow \mathrm{W}_{\text{time}}(t_1, t_3))$

Parts determine times

$\forall t_1\, \forall t_2\, ((t_1 \equiv_{\text{time}} t_2) \leftrightarrow \forall t_3\, (\mathrm{W}_{\text{time}}(t_3, t_1) \leftrightarrow \mathrm{W}_{\text{time}}(t_3, t_2)))$

$<_{\text{time}}$ *determines an ordering*

$\forall t_1 \neg (t_1 <_{\text{time}} t_1)$

$\forall t_1\, \forall t_2\, \forall t_3\, ((t_1 <_{\text{time}} t_2) \wedge (t_2 <_{\text{time}} t_3) \rightarrow (t_1 <_{\text{time}} t_3))$

Parts and wholes are unrelated in the ordering

$\forall t_1\, \forall t_2\, (\mathrm{W}_{\text{time}}(t_1, t_2) \rightarrow \neg (t_1 <_{\text{time}} t_2) \wedge \neg (t_2 <_{\text{time}} t_1))$

Parts of times are related to other times in the ordering as the whole is related

$$\forall t_1\ \forall t_2\ \forall t_3\ (\mathrm{W}_{\mathrm{time}}(t_1, t_2) \wedge (t_2 <_{\mathrm{time}} t_3)) \rightarrow (t_2 <_{\mathrm{time}} t_3))$$

$$\forall t_1\ \forall t_2\ \forall t_3\ (\mathrm{W}_{\mathrm{time}}(t_1, t_2) \wedge (t_3 <_{\mathrm{time}} t_3)) \rightarrow (t_3 <_{\mathrm{time}} t_1))$$

Times are intervals

$$\forall t_1\ \forall t_2\ \forall t_3\ (\mathrm{W}_{\mathrm{time}}(t_1, t_3) \wedge \mathrm{W}_{\mathrm{time}}(t_2, t_3) \rightarrow \mathrm{W}_{\mathrm{time}}(t_2, t_1) \vee \mathrm{W}_{\mathrm{time}}(t_1, t_2) \vee \mathrm{X}_{<}(t_1, t_2) \vee \mathrm{X}_{<}(t_2, t_1) \vee (t_1 <_{\mathrm{time}} t_2) \vee (t_2 <_{\mathrm{time}} t_1))$$

$$\forall t_1\ \forall t_2\ \forall t_3\ (\mathrm{W}_{\mathrm{time}}(t_2, t_1) \wedge \mathrm{W}_{\mathrm{time}}(t_3, t_1) \wedge (t_2 <_{\mathrm{time}} t_3) \rightarrow \forall t_4\,((t_2 <_{\mathrm{time}} t_4 <_{\mathrm{time}} t_3) \rightarrow \mathrm{W}_{\mathrm{time}}(t_4, t_1)))$$

Overlapping times are not related in the ordering

$$\forall t_1\ \forall t_2\ (\mathrm{X}_{<}(t_1, t_2) \rightarrow (\neg (t_1 <_{\mathrm{time}} t_1) \wedge \neg (t_2 <_{\mathrm{time}} t_1)))$$

Axioms for locations

$\mathrm{W}_{\mathrm{time}}$ *is a part-whole relation*

$$\forall l\ \mathrm{W}_{\mathrm{time}}(l, l)$$

$$\forall l\ \forall p\ (\mathrm{W}_{\mathrm{time}}(l, p) \wedge \mathrm{W}_{\mathrm{time}}(p, l) \rightarrow (l \equiv_{\mathrm{location}} p))$$

$$\forall l_1\ \forall l_2\ \forall l_3\ (\mathrm{W}_{\mathrm{time}}(l_1, l_2) \wedge \mathrm{W}_{\mathrm{time}}(l_2, l_3) \rightarrow \mathrm{W}_{\mathrm{time}}(l_1, l_3))$$

Parts determine locations

$$\forall l_1\ \forall l_2\ ((l_1 \equiv_{\mathrm{location}} l_2) \leftrightarrow \forall l_3\,(\mathrm{W}_{\mathrm{location}}(l_3, l_1) \leftrightarrow \mathrm{W}_{\mathrm{location}}(l_3, l_2)))$$

Axioms for truth related to time and location

Outward closure of truth for locations

$$\forall\ t\ \forall l\,(\mathrm{E}(t, l) \rightarrow (\forall p\,(\mathrm{W}(l, p) \rightarrow \mathrm{E}(t, p))))$$

Downward and upward closure of truth in time at a location

$$\forall\ t\ \forall l\,(\mathrm{E}(t, l) \leftrightarrow (\forall w\ \mathrm{W}_{\mathrm{time}}(w, t) \rightarrow \mathrm{E}(w, l))))$$

Mass-process axioms and contexts

Subordination and truth in a context

$$\forall \ldots (\mathrm{E}\ \mathrm{sub}\ \mathrm{F}) \rightarrow \forall t\ \forall l\,(\mathrm{E}(t, l) \rightarrow \mathrm{F}(t, l))$$

Substitution of equivalent categorematic words

$$\forall \ldots (\mathrm{E} \approx \mathrm{F} \rightarrow (\mathrm{A}(\mathrm{F}) \leftrightarrow \mathrm{A}(\mathrm{E})))$$

where A(E) is an atomic proposition in which E appears, and A(F) is A(E) with F replacing some but not necessarily all occurrences of E.

Non-describing local categorematic words are nil

$\forall \ldots\ (\neg \mathrm{E}(t, l) \rightarrow\ \neg \mathrm{A})$

where A is an atomic proposition in which $[\![\underline{\text{this}}\ \mathrm{E}(t, l)]\!]$ appears.

Describing local words are true of their time and location

a. $\forall \ldots (\mathrm{E}(t, l) \rightarrow\ [\![\underline{\text{this}}\ \mathrm{E}(t, l)]\!]\ (t, l))$

b. $\forall \ldots ([\![\underline{\text{this}}\ \mathrm{E}(t, l)]\!]\ (w, p) \rightarrow\ \mathrm{E}(w, p))$

Continuity of description in time (*including names*)

$\forall \ldots\ (\mathrm{E}(w_1, p_1) \wedge\ \mathrm{E}(w_2, p_2) \wedge\ w_1 <_{\text{time}} w_3 <_{\text{time}} w_2$
$\rightarrow\ \exists w_3\, \mathrm{E}(w_3, p_3))$

where E is any formal local categorematic word or name symbol

Subordination of a local categorematic word to the categorematic word on which it is based

$\forall \ldots\ (\mathrm{E}(t, l) \rightarrow\ ([\![\text{this}\ \mathrm{E}(t, l)]\!]\ \text{sub}\ \mathrm{E}))$

Subordination yields subordination of the local categorematic words

$\forall \ldots\ (\mathrm{E}\ \text{sub}\ \mathrm{F} \wedge\ \mathrm{E}(t, l) \rightarrow\ ([\![\text{this}\ \mathrm{E}(t, l)]\!]\ \text{sub}\ [\![\text{this}\ \mathrm{F}(t, l)]\!])$

Identity and equivalence

$\forall \ldots\ (\mathrm{E} \approx \mathrm{F} \wedge\ [\![\text{this}\ \mathrm{E}(t, l)]\!]\ (w, p) \wedge\ [\![\text{this}\ \mathrm{F}(w, p)]\!]\ (t, l),$
$\rightarrow\ [\![\text{this}\ \mathrm{E}(t, l)]\!] \approx [\![\text{this}\ \mathrm{F}(w, p)]\!])$

Iterations of local categorematic words

$\forall \ldots\ ([\![\underline{\text{this}}\ \mathrm{E}(t, l)]\!]\ (w, p) \rightarrow$
$[\![\text{this}\ [\![\text{this}\ \mathrm{E}(t, l)]\!]\ (w, p)]\!] \approx [\![\text{this}\ \mathrm{E}(t, l)]\!])$

Disjoint locations at the same time yield different references

$\forall \ldots\ ([\![\text{this}\ \mathrm{E}(t, l_1)]\!] \approx [\![\text{this}\ \mathrm{E}(t, l_2)]\!] \rightarrow$
$\exists p\, (\mathrm{W}_{\text{location}}(p, l_1) \wedge\ \mathrm{W}_{\text{location}}(p, l_2) \wedge$
$[\![\text{this}\ \mathrm{E}(t, p)]\!] \approx [\![\text{this}\ \mathrm{E}(t, l_1)]\!])$

A name cannot be true at some time in disjoint locations

$\forall \ldots\ (\mathrm{N}(t, l_1) \wedge\ \mathrm{N}(t, l_2) \rightarrow$
$\exists l_3\, (\mathrm{W}_{\text{location}}(l_3, l_1) \wedge\ \mathrm{W}_{\text{location}}(l_3, l_2) \wedge\ \mathrm{N}(t, l_3))$

Subordination is reflexive

a. E sub E

if E is an ordinary formal categorematic word

b. $\forall \ldots\ (\mathrm{E}(t, l) \rightarrow\ [\![\underline{\text{this}}\ \mathrm{E}(t, l)]\!]\ \text{sub}\ [\![\underline{\text{this}}\ \mathrm{E}(t, l)]\!])$

Subordination is transitive

$\forall \ldots ((E_1 \text{ sub } E_2) \wedge (E_2 \text{ sub } E_3) \rightarrow (E_1 \text{ sub } E_3))$

Commutativity of conjuncts in a conjunction

$\forall \ldots A(F) \leftrightarrow A(F')$

where F is a conjunction of categorematic words; E_1 and E_2 are conjuncts in F; and F′ is F with E_1 and E_2 replacing each other; and A(F) is an atomic proposition in which F appears; and A(F′) is A(F) with some but not necessarily all occurrences of F replaced with F′.

Subordination of a conjunction to its conjuncts

$\forall \ldots (E \text{ sub } E) \rightarrow (E \text{ sub } F)$

where E is a conjunction of categorematic words, and F is E with one or more of its conjuncts deleted.

Conjunction preserves subordination

$\forall \ldots [((E_1 \text{ sub } E_2) \wedge (F \text{ sub } F)) \rightarrow ((F + E_1) \text{ sub } (F + E_2))]$

Subordination of a modified categorematic word

$\forall \ldots ((E / F) \text{ sub } (E / F) \rightarrow (E / F) \text{ sub } E)$

Modifying a categorematic word by itself adds nothing

$\forall \ldots (E \text{ sub } E) \rightarrow (E / E \approx E)$

Unity of names

$\forall \ldots N(t, l) \rightarrow [\![\underline{\text{this}} \ N(t, l)]\!] \approx N$

The definitions of theorem and valid formal inference are the usual ones (Volume 0).

The logical axioms here are those for two-sorted classical predicate logic, and so the axiom system is strongly complete.[18] But, as in the remarks following the axiom system **MPSub**, there is no reason to think that all formal wffs that are true relative to our informal conceptions of mass-process, subordination, relating, reference, identity, and names are theorems of this logic.

[18] See *Classical Mathematical Logic.*

24 Examples of Formalizing

Example 1 Suzy (pointing, on April 29, 2010): *This dog existed three years ago.*

Analysis This is thing-talk. All we can say in our mass-process language is that the dog-ing here is the same as three years ago:

$$\exists\, l\, (\, [\![\text{ this DOG (April 29 2010, here) }]\!] \approx [\![\text{ this DOG (April 29 2007, } l) \,]\!] \,)$$

But this would be true whether "DOG (April 29 2010, here)" is a correct description of one dog or many dogs, dead or alive, or served on a platter.

Example 2 The dog barked.

Analysis It might seem that we can formalize this with:

$$\exists t\, \exists l\, (\, (\text{DOG} + \text{BARK})\, (t, l) \wedge\ (t <_{\text{time}} p) \,)$$

But this would be true if there were several dogs barking. The word "the" in the example is meant to indicate only one dog, the reference of which is given by context in ordinary speech. Perhaps that context could provide a way to formulate a this-word to use here.

Example 3 The waves crashed on the shore.

Analysis In process-mass logic we can describe the scene we imagine the example is meant to describe:

$$\exists t\, \exists l\, (\, [\, (\text{WATER} + \text{WAVE} + \text{CRASH})\, (t, l)\ \underline{\text{on}}\ (\text{SHORE})\, (t, l)\,] \wedge\ (t <_{\text{time}} p) \,)$$

This would serve also to describe what we might say in English as "Some waves crashed on the shore" or "Some waves crashed on a shore". The use of the first "the" is a dummy word, not meant to indicate a single thing but used only as a variant way to say "Waves crashed on the shore". The second is used to point our attention to the land that is picked out by context in ordinary speech, but we need not take it into consideration here for it treats a shore as a thing.

Waves in both our common and scientific thought are mass-process, not things. Yet in English we talk of waves and speak of the first wave, the second wave. Describing waves in the flow of all, we can talk of one versus another only by talking of one time and location of the flux versus another. We see water rising and crashing on the shore, then rising and crashing on the shore again and again. We could talk of crashings as much as waves. A wave is a particular rising of the water, which we can single out only by singling out the time and place of that rising and water. So we can describe the first wave crashing on the shore, after, say, 3:00 p.m. April 19, 2010 as:

$\exists t\, \exists l$ ((WATER + WAVE + CRASH) (t, l) <u>on</u> (SHORE) (t, l)
$\wedge$ (3:00 p.m. April 19 2010 $<_{\text{time}}$ t))
$\wedge$ $\neg$ $\exists t_1$ ((WATER + WAVE + CRASH) (t_1, l) <u>on</u>
(SHORE) $(t_1, l) \wedge$ (3:00 p.m. April 19, 2010 $<_{\text{time}}$ t_1 $<_{\text{time}}$ t))

Example 4 If we take some ice cubes from the refrigerator, crush them, and put them into a glass of coke, we may say:

The ice in the coke is the same ice that was in the refrigerator before.[19]

Analysis We can assert the identity of the ice in the form of cubes with the ice that is crushed in the glass. With appropriate references for t_1, l_1 and t_2, l_2, where the time for the former is before that of the latter, we could say:

$[\![$ this ICE (t_1, l_1) $]\!]$ $\approx$ $[\![$ this ICE (t_2, l_2) $]\!]$

In contrast,

$[\![$ this (ICE + CUBE) (t_1, l_1) $]\!]$ is describing

$[\![$ this (ICE + CUBE) (t_2, l_2) $]\!]$ is not describing

If at a later time t_3 the variable l_3 refers to a location where the glass is, and the ice in it has partially melted, then:

$\neg$ ($[\![$ this ICE (t_2, l_2) $]\!]$ $\approx$ $[\![$ this ICE (t_3, l_3) $]\!]$)

Example 5 On the table there is a gold ring, a gold pin, and a gold coin.

Analysis With appropriate references for the variables, we might use:

(GOLD + RING) (t, l_1) <u>on</u> (TABLE) (t, l)
$\wedge$ (GOLD + PIN) (t, l_2) <u>on</u> (TABLE) (t, l)
$\wedge$ (GOLD + COIN) (t, l_3) <u>on</u> (TABLE) (t, l)

But this misses the uses of “a” and “the”. We could, though, distinguish the ring-ing and the pin-ing and the coin-ing by saying they are in different locations:

(GOLD + RING) (t, l_1) <u>on</u> (TABLE) (t, l_4)
$\wedge$ (GOLD + PIN) (t, l_2) <u>on</u> (TABLE) (t, l_4)
$\wedge$ (GOLD + COIN) (t, l_3) <u>on</u> (TABLE) (t, l_4)
$\wedge$ $\neg$ $\exists$ l ($\text{W}_{\text{location}}(l, l_1) \wedge$ $\text{W}_{\text{location}}(l, l_2)$)
$\wedge$ $\neg$ $\exists$ l ($\text{W}_{\text{location}}(l, l_2) \wedge$ $\text{W}_{\text{location}}(l, l_3)$)
$\wedge$ $\neg$ $\exists$ l ($\text{W}_{\text{location}}(l, l_3) \wedge$ $\text{W}_{\text{location}}(l, l_1)$)

If the reference for l is a small enough location, we could pick out the “totality” of gold on the table with:

(a) $[\![$ this GOLD (t, l) $]\!]$

[19] From Harry C. Bunt, *Mass Terms and Model Theoretic Semantics*, p. 36.

But this could be true if there were also a gold toy figurine on the table. Even saying that there is no gold-ing going on at time *t* within the location l_4 except within locations l_1, l_2, and l_3 won't exclude that.

To accept the gold ring-ing, the gold pin-ing, and the gold coin-ing as parts of what (a) refers to is, according to Alice ter Meulen in *Substances, Quantities, and Individuals,* to view the gold on the table as a whole even though it is not connected.

> In some respects quantities of stuff are quite like individuals. Individuals are objects in space-time, and so are all quantities of stuff. They are in this sense part of the same physical reality. . . . But quantities of substances are in many other respects to be distinguished from individuals. The first most striking difference between quantities and individuals is the fact that the quantities of any substance can be divided into smaller parts that are also quantities of the same substance. Similarly, the quantities of some substance can become part of a larger quantity of the same substance. . . . The fact that quantities can be divided into quantities of the same substance together with the fact that any number of quantities of some substance can become part of a new quantity of the same substance is a logical property characteristic of quantities only. This property of quantities, called the *property of homogeneous reference*, has widely been recognized as distinctive of the semantic interpretation of mass terms. A more precise formulation of this property is the following. Any parts of a quantity of x that are themselves quantities of x can become parts of another quantity of x. pp. 67–68 [20]

But in the view of the world as process, the gold-ing at that time and place is not a whole: there is only this gold-ing and that gold-ing, some more extensive than others, all subsumed under the description "GOLD" but not part of what is described by that.

Example 6 What Kim spilled is the same coffee as Sandy wiped up.[21]

Analysis We can formalize this as:

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[20] In contrast, Harry C. Bunt in *Mass Terms and Model Theoretic Semantics* views a homogeneous reference principle as an observation about how we talk and not about the nature of the world:

> I therefore hold the view that a linguistic semantic theory should take into account that the use of a mass noun forms a way of speaking about things as if they were homogeneous masses, that is, as having some internal structure, allowing us to refer to certain parts, but without singling out any particular parts and without any commitments concerning the existence of minimal parts. I call this assumption the *homogeneous reference hypothesis*, and formulate it for convenience as follows:
>
> > Mass-nouns refer to entities as having a part-whole structure without singling out any particular parts and without making any commitments concerning the existence of minimal parts.
>
> I believe that this hypothesis . . . expresses in what way mass nouns are semantically different from count nouns. The difference is not in the structure of the entities that mass nouns and count nouns refer to, but in the way in which they refer to these entities. p. 46

[21] The example comes from "Mass Expressions" by F. J. Pelletier and L. K. Schubert, p. 359.

$\exists t_1 \exists t_2 \exists l_1 \exists l_2$

$[\ (\text{(KIM + SPILL)}\ (t_1, l_1)\ \underline{\text{directed towards}}\ \text{(COFFEE}\ (t_1, l_1))$

$\wedge\ (\text{(SANDY + WIPE)}\ (t_2, l_1)\ \underline{\text{directed towards}}\ \text{(COFFEE}\ (t_2, l_1))$

$\wedge\ (\llbracket \text{this COFFEE}\ (t_1, l_1) \rrbracket \approx \llbracket \text{this COFFEE}\ (t_2, l_2) \rrbracket)\ ]$

We might replace the last conjunction with:

$\llbracket \text{this COFFEE}\ (t_1, l_1) \rrbracket\ (t_2, l_2)$

I'll leave to you to puzzle out which best formalizes "the same" in the example.

Example 7 Wanda used to be thin.

Analysis In thing-talk we conceive of things changing: a predicate that once applied to Wanda no longer applies to her. Taking "WANDA" as a name, we can formalize the example as:

$\exists t \exists l_1 \exists l_2\ \text{(WANDA + THIN)}\ (t, l_1) \wedge\ (t <_{\text{time}} \text{now})$

$\wedge\ \text{WANDA (now}, l_2) \wedge\ \neg\ \text{(WANDA + THIN) (now}, l_2))$

Example 8 This mud used to be brown.

Analysis It is not only things that change. With appropriate reference for *l*, we can use the following to pick out the mud we are talking about:

$\llbracket \text{this MUD (now}, l_1) \rrbracket$

Then we can formalize the example in process-mass logic as:

$\exists t \exists l_1 \exists l_2\ (\ (\llbracket \text{this MUD (now}, l_1) \rrbracket + \text{BROWN})\ (t, l_2) \wedge\ (t <_{\text{time}} \text{now})$

$\wedge\ \neg\ (\llbracket \text{this MUD (now}, l_1) \rrbracket + \text{BROWN) (now}, l_1))$

We need l_2 here because the mud could have been in a place in a yard and then scooped up and carried to make an adobe brick in another place in that yard. Note that we don't need in to add a conjunct "$\llbracket$ this MUD (now, l_1) $\rrbracket$ (now, l_1)" because that follows from the first conjunct.

Example 9 This running used to be fast.

Analysis What we conceive of as process in our ordinary talk can change, too: Dick could have started running fast and slowed down after 20 meters . We can formalize the example in process-mass logic as:

$\exists t \exists l_1 \exists l_2\ (\ (\llbracket \text{this RUN (now}, l_1) \rrbracket + \text{FAST})\ (t, l_2) \wedge\ (t <_{\text{time}} \text{now})$

$\wedge\ \neg\ (\llbracket \text{this RUN (now}, l_1) \rrbracket + \text{FAST) (now}, l_1))$

Or at least we can if we take "FAST" to be a categorematic word.

Example 10 All is change.

Analysis We can formulate talk of what we can refer to with a this-word as

changing relative to a particular description, as we did in the last three examples. If we were to allow quantifications over wffs used as descriptions we could even say that whatever we can refer to at any two times changes relative to some description. But change as endemic, change as the nature of flux, is incoherent, for there is no *that* to change. Even the word "process" is misleading, for it suggests change and instability, when there is nothing to be unstable. There is only the flux and parts of it talked about under specific descriptions to which further descriptions can apply or not apply at different times and places. It would be as apt to describe the world as stuff, as some translators of ancient Greek philosophy do.[22]

Example 11 Dick and Zoe are walking in the forest. Dick says to Zoe:

The river is very deep here. Don't step into it.

Analysis Dick is thinking of the river as a thing when he says "the river". But rivers are changing, flowing. Zoe, getting the hang of thinking of the world as process, quotes Heraclitus:

You can't step into the same river twice.

Then Tom, who is along with them, butts in:

River? What river?

He's right from the process-mass view that a river is not a thing. But we can talk of the river with a description of a part of the flow of all:

$[\![$ this RIVER (now, l) $]\!]$

With appropriate reference for l this refers to the river-ing in front of Dick and Zoe and Tom. We can even talk of stepping into that once (with appropriate references for the variables):

(ZOE + STEP) (t_1, l_1) <u>into</u> $[\![$ this RIVER (now, l) $]\!]$

or twice:

(ZOE + STEP) (t_1, l_1) <u>into</u> $[\![$ this RIVER (now, l) $]\!]$

$\wedge$ (ZOE + STEP) (t_2, l_2) <u>into</u> $[\![$ this RIVER (now, l) $]\!]$

$\wedge \neg$ W (t_1, t_2) $\wedge \neg$ W (t_2, t_1)

or as many times as we wish.

Example 12 Dick had the same idea as Suzy.

Analysis Ideas need not be things for us to assert identity. We can formalize the example as:

$\exists t_1 \exists l_1 \exists t_2 \exists l_2$ ((SUZY + IDEA) (t_1, l_1)

$\wedge$ (DICK + IDEA) (t_2, l_2) $\wedge$ ($t_1 <_{\text{time}}$ now) $\wedge$ ($t_2 <_{\text{time}}$ now)

$\wedge$ ($[\![$ this IDEA (t_1, l_1) $]\!] \approx [\![$ this IDEA (t_2, l_2) $]\!]$)

[22] See B.A.G. Fuller and Sterling M. McMurrin, *A History of Philosophy*.

Example 13 Once a dog, always a dog.

Analysis In *Time and Space in Formal Logic* this example was viewed as an assertion that being a dog is an essential attribute of a thing in time. In the view of the world as process-mass there are no things and no attributes of things.

We might think to formalize the example by saying that any dog-ing—whether of one or many, dead or alive—continues to be dog-ing at any other time. To say that, we would have to use "⟦ this DOG (*t*, *l*) ⟧" with some references for *t* and *l*, and that already has continuity over time. What we can't say is that what once was a dog could now be a cat, for there is no thing, no object, on which to hang the descriptions "DOG" and "CAT".

Example 14 Dick's mind is not the same as Dick's body.

Analysis The idea that the mind and body are distinct entities can't be said in our mass-process language. There is the flow of all, which in part can be described as body-ing and in part can be described as mind-ing, and a description of some of the flow perhaps might require both those terms, but not conceived as two things, as separate existences.

Can we formulate some version of the mind-body problem in the language of process-mass logic? If a mind is not a physical thing and a body is, then Cartesian dualism is certainly true. But if mind is abstract we could not reason about it in process-mass logic. But we can assert that mind-ing is not body-ing,

¬ (MIND ≈ BODY)

and Dick's mind-ing is not equivalent to Dick's body-ing:

¬ [(DICK + MIND) ≈ (DICK + BODY)]

This could be true even if in every context in which it would be correct to assert "DICK + MIND" it would be correct to assert "DICK + BODY" and vice-versa, for subordination cannot be reduced to truth in context.

Example 15 Richard L. Epstein wrote Predicate Logic.

Analysis In thing-talk we have difficulty being clear about what we mean by a book. Is it a particular inscription? Is it a piece of language, perhaps an ordering of words? Is it a concrete thing or something abstract?

In process-mass logic we can use "*Predicate Logic*" as a categorematic word. Then we can say of a particular time and place:

(a) *Predicate Logic* (*t*, *l*)

If our conception of the book in thing-talk is that there is one and only one such book, then it is not a physical thing. But we can still use (a): at a place and time there could be *Predicate Logic*-ing in many copies or in the mind of someone memorized completely, as we talk of the *Odyssey* when there were no copies of it.

For any reference for the variables *t* and *l*, we should have:

⟦this *Predicate Logic*(*t*, *l*)⟧ ≈ *Predicate Logic*

That's what we get by taking "*Predicate Logic*" as a name.

But what if all copies of the book were destroyed, all record of it destroyed, yet someone, without realizing it, committed it to memory and later when asked to write it down, transcribed the words and symbols in exactly the right order, as we sometimes see with someone who can write down the words of a song when asked even though she never thought she'd memorized it. It would be a stretch to say that the book existed in the unconscious memory of that person. Whether to use "*Predicate Logic*" as a name depends on how we conceive of books.

Example 16 Zeus is Jupiter.

Zeus ≈ Jupiter

Analysis We can take this as true even if, as I suspect most of us believe, there is no time and place at which either word is a correct description of the flow of all. (Compare Example 6 of Chapter 10.)

Example 17 Pegasus is Bellerophon's horse.

Analysis This seems to be the kind of predication we can formalize, using subordination. We can point and say "horse of Bellerophon" with:

(a) HORSE of BELLEROPHON

But we can't use "PEGASUS sub (HORSE of BELLEROPHON)" because (a) is not a categorematic word. Perhaps someone will see a way to treat such a combination as one.

Example 18 Sherlock Holmes was a detective. He could have been a professor.

Analysis We can't formalize this not because "Sherlock Holmes" is, as we would say in thing-talk, a non-referring name. It's because we can't formalize this kind of predication (see Example 1 of Chapter 14).

One colleague said that if we can't formalize this in our mass-process language and logic, then we can't analyze fiction. But that is to take "fiction" to mean talk about things. Can't we have also fictional mud and fictional running? In any case, even in classical predicate logic we can't formalize the example because of the modal "could". All we can do is set up one model in which Sherlock Holmes was a detective and another in which he was a professor (see Chapter 41 of *The Internal Structure of Predicates and Names*). Those who try to formalize the two sentences together in one language with one model are trying to combine the metalogic and the logic into one system, and that does not work.[23]

[23] See my "Reflections on Temporal and Modal Logic".

Example 19 There are two patches of mud now in the patio.

$$\exists l_1 \exists l_2 \; [\, \text{MUD}(\text{now}, l_1) \wedge \text{MUD}(\text{now}, l_2) \wedge \text{W}_{\text{location}}(l_1, \text{patio}) \wedge \text{W}_{\text{location}}(l_2, \text{patio}) \wedge \neg \exists l_3 \, (\text{W}_{\text{location}}(l_3, l_1) \wedge \text{W}_{\text{location}}(l_3, l_2)) \,]$$

Analysis As a consequence of this we have:

$$\neg \, ([\![\text{this MUD}(\text{now}, l_1)]\!] \approx [\![\text{this MUD}(\text{now}, l_2)]\!])$$

The word “patches” is a classifier, making mud-ings into things. We don’t talk about things here. Rather, the formalization is apt for “There are two mud-ings now in the patio”. We can count mud-ings. Doesn’t that show we have a notion of thing in our language and logic? We can count because we treat times and locations as things that we can quantify over. But even that does not make two mud-ings into two things. We can talk about two dog-ings, but those could be a dead dog and a pack of dogs. That we can count does not by itself show that we have a usable notion of individual thing in our language and logic.

25 Talking of Mass-Process and Things Together

Talking of things in our mass-process language and logic?
In English we use particular words or phrases to pay attention to a part of a mass as a thing: “a glass of water”, “a patch of mud”, and also “three patches of mud”, “four cups of coffee”. Such phrases are called *classifiers.* The most general, all-purpose classifier in English is “a bit of”, though that usually suggests a small portion. So we can say “a bit of water”, “a bit of chocolate”, “a bit of gold”. However, this sounds more than odd when we use it with a mass word that isn’t, from our English-speaking perspective, a substance: “a bit of justice”, “a bit of honor”. For those we use a different general classifier: “an instance of justice”, “an instance of honor”.

Chinese has classifier words. As Perry Link says in “A Magician of Chinese Poetry”:

> Nouns have no number in Chinese. Weinberger notes that “a rose is a rose is all roses,” but that formulation still leaves us too far inside Western-language number habits. “All roses” in English means the summation of individual roses, whereas in Chinese *meigui*, or “rose” is more like “roseness” or “rosehood.” (If you want to talk in Chinese about one rose, you may, but then you use a “measure-word” to say “one blossom-of roseness.”) p. 50

Dorothy Lee in “Categories of the Generic and Particular in Wintu”, says that in Wintu “particularization” is done by adding a suffix to a “generic” word.

> Particularizations may be used to create a new word, denoting a delimited form of something commonly regarded as generic. p. 366

But when you particularize “dog” in Wintu, that doesn’t mean “a dog”. The classifier indicates specific portions of the mass, but is neither singular nor plural. Lee says that the word *nop* means “deer” in the general mass-process sense, and *nopum* means what we would translate as “one or several deer”. This is very much like what we do with the this-operator in our mass-process language and logic. We particularize, but we do not individualize; we describe but necessarily an individual.

And that’s the problem with adding a general classifier to our mass-process language and logic. We could add the predicate “— is a bit of (DOG)”, but what could we put into the universe of individuals for a model that would satisfy this? A single dog? Several dogs? A pack of dogs? A portion of dog meat? We could have the predicate “— is a bit of (SNOW)”, but what bits of snow should be in the universe of a model in reasoning about clearing snow from city streets? We can’t talk about all bits of snow any more than we can talk about all things: it’s simply not clear what counts as a bit of snow. Can I identify and re-identify a fall of snow? Can I identify and re-identify a drift of snow? Is a collection of snow flakes 3 cm in diameter that is embedded in that drift a bit of snow? Is a collection of flakes that is

falling just now a bit of snow? To use this thing-in-mass language and logic, we would have to specify what bits of snow, what bits of water, what bits of mud are in the universe of a model. Since snow, water, and mud do not come naturally in bits, a general way to specify what is in the model eludes us. And that's just for mass-process words that are about "substance". What counts as a bit of justice? How can I identify and re-identify some thing that *x* can stand for that makes "(— is a bit of JUSTICE) (*x*)" true?

We can have a notion of a single thing that is "part of" a mass only if we start with that notion from our thing-talk using classifiers that we agree are meant to pick out what we can reason about as things: "a ball of snow", "a drift of snow", "a single dog", "a pack of dogs", "a portion of dog meat". We cannot find in a mass-process language a hidden or implicit notion of thing of the sort that we could use in predicate logic. We have to import it from our thing-conception world. The best we can do is use many different classifiers.

If we do add the predicate "— is a patch of (MUD)" to a realization of our mass-process language, we need to relate it to "MUD". Perhaps we could treat it as if it were a mass-process word, too, so we could have "— is a patch of (MUD) sub MUD". But a predicate is a piece of language, and we would have to also treat it as picking out a concept for that to be right. Otherwise we should have:

"— is a patch of (MUD)" sub "MUD"

I do not see how to proceed here. We should talk with native speakers of mass-process languages who also speak English and ask them how they can use the notion of a thing that they have learned in English in their language. What tools do they use? Can we formalize those?

Talking of mass-process in predicate logic?

Perhaps we could talk about mass-process and things together by starting instead with classical predicate logic. In English we can turn the common noun "father" into a mass-process word, "fatherhood". We can turn the adjective "wise" into a mass-process word, "wisdom". We can turn the verb "run" into a mass-process word, "running". We don't have a regular procedure for making mass words. We often have to make up a word when we want to "massify" a common noun, for example "doggieness" or "doghoodity".

Some people who are committed to a thing-view of the world say that we do have a general method: just add "the property of", as in "the property of being wise" or "the property of being a dog". But that is not what "wisdom" means nor what "doggieness" is meant to convey. "Wisdom" is not meant as the property of a thing to be wise; it is a mass term. Doggieness is not the property of a thing to be a dog; it is a mass term as much as "DOG" is in our mass-process language. Even as English speakers, we think of doggieness as the essence of being a dog, or as the genus of

dog, not as a property. Running, too, is not a property of a thing that is running; it is a mass term as much as "RUN" is in our mass-process language.

To incorporate mass-process talk into predicate logic, perhaps we could use a general way to massify common nouns, verbs, and (some) adjectives with a logical operator *mass*. From "— is a dog", "— runs", "— is white" we would get "mass (— is a dog)", "mass (— runs)", and "mass (— is white)". Given any predicate P, we would have a mass term "mass (P)". We could include in the language "sub" for the subordination relation, so that we could assert "mass ((— is a human)) sub mass ((— is an animal))".

We would also need to include as primitive in the vocabulary some mass words such as "mud", "water", and "snow" that are not meant to be used as predicates. But then we have all the problems we saw in trying to put a notion of thing into our mass-process language and logic. Only here we need not look for a speaker of a mass-process language to help us. We have all the data we need because we have mass-process words in English, and we do individuate with them with classifiers.

There is no clear path to dealing with things and mass-process in a formal logic based on predicate logic, though some have tried. Nor is there a clear path to talking about things and mass-process in our logic of mass-process.

Aside: Individuating mass terms

Harry C. Bunt in *Mass Terms and Model Theoretic Semantics* gives Example 4 of Chapter 24 above about ice in his criticism of the view of masses as collections of things. He says:

> The choice of an appropriate "individuating standard" must depend on the circumstances; for example, "sugar" will have to be counted as "lumps of sugar" in some contexts, as "grains of sugar" in other contexts, and as "shipments of sugar" in still other contexts. This context-dependence alone makes the proposal [to use sets as the denotation of mass terms] rather unattractive; moreover, it runs into fundamental difficulties, illustrated by [that example]. [It] would be a true sentence about some ice, yet there is no individuating standard in terms of which we can express this, since the identity stated by the sentence is not an identity of any of the pieces of ice involved, but an identity of the totalities of ice made up of whatever pieces are involved.
>
> If context-dependent individuating standards do not work satisfactorily, the next move is naturally to look for context-independent individuating standards. Such standards would then have to be artificial, since we just saw that natural standards, suggested by the language ("dollop", "lump", "batch", etc.) do not work in general. It is tempting to think of Quine's minimal parts hypothesis and treat mass nouns as denoting the sets of their minimal parts. "Water" would denote the set of H_2O molecules, "furniture" the set of chairs, tables, etc., and "sugar" the set of sugar grains. I have not seen any serious proposal for such an approach, though it would seem to encounter fewer formal difficulties than the use of context-dependent individuating standards. Presumably, this is due to the fact that such a proposal would so obviously run counter to our intuitions. Even if one were to agree with the minimal parts hypothesis, it is often impossible to actually determine a reasonable set of minimal parts. To consider H_2O

> molecules as the referents of “water” seems counter-intuitive; something like “drops” would seem better, but presents the problem that a drop can be split into smaller drops, so it clearly is not really a minimal part, and the same is true of any other part we can name without making use of technical terms from physics or chemistry. Moreover, for abstract nouns like “leisure:, “damage”, or “time”, for which no minimal parts are assumed to exist, this proposal must fail. pp. 36–37

See my “Models and Theories” for a fuller explanation of why it is wrong to consider water as a collection of H_2O molecules.

Contexts via Before and After

26 Categorematic Words in Time

Viewing times and locations as things is not an essential part of our conceptions of time and space. More fundamental at least for time are comparisons of before and after. "Spot barked before Dick yelled" orients us, to some extent, in the mass of time. No thing-talk is needed or implicit in such a comparison: we do not talk of events; we join propositions. In Volume 2 we saw how to formalize talk of before and after using temporal propositional connectives.

Adding the logic of temporal propositional connectives to our base logic of mass-process can give us contexts without talk of bits of time. If I say "DICK + YELL", I can add "before DOG + BARK", and "after CAT + MEOW". This gives us a context in time. I have found no way to compare and to orient ourselves in space with spatial propositional connectives. So adding the logic of temporal propositional connectives to the logic of mass-process is only a partial solution to providing contexts in the manner of our earlier examples. But then, giving contexts in terms of time and space is only a partial solution, for there is always more we could take account of in describing context.

* * * * * * * * *

We begin with the language and logic of temporal propositional connectives, **TC** of *Time and Space in Formal Logic*. Each atomic proposition, such as "Spot barked", is not of a time but if true establishes a time: when Spot barked. We focus better on that time by relating it to the time that "Dick yelled" establishes as well as times that other true atomic propositions establish. The time that the proposition "Spot barked" establishes is an interval: before that interval Spot didn't bark, after it he didn't bark, and during the whole interval he barked. Intervals have the same status to the mass of time as some mud in my patio has to the mass-process mud.

We use the following connectives of atomic propositions for before and after relations, where I'll abbreviate "the time that p establishes" as "the time of p":

$p \wedge_{bb} q$ is true iff both p and q are true and the time of p begins before the beginning of the time of q.

$p \wedge_{eb} q$ is true iff both p and q are true and the time of p ends before the beginning of the time that q establishes.

$p \wedge_{be} q$ is true iff both p and q are true and the time of p begins before the ending of the time that q establishes.

$p \wedge_{ee} q$ is true iff both p and q are true and the time of p ends before the ending of the time that q establishes.

In Volume 2 we used these connectives by designating or looking for beginnings and endings of, for example, when Spot barked and of when Dick yelled in

order to compare. But talk of beginning and ending points, I suggested there, is not essential to the logic. All that is essential is that there is a before and after of intervals of times of propositions in terms of overlaps. The intervals are not things but only some of the flow of time we note, as we note this lump of snow.

A categorematic word, such as "DOG", can be a correct description not at a time but within time. Pointing, I say "DOG + SLEEP" and then a little later I say "DOG + RUN". Those are true: there was dog-ing together with sleeping before there was dog-ing together with running. This suggests the formalization:

$$(\text{DOG} + \text{SLEEP}) \wedge_{\text{eb}} (\text{DOG} + \text{RUN})$$

But suppose later I point and say "DOG + SLEEP" again. That looks like the same proposition I said earlier. But it's meant to be a different one, talking of after the time that "DOG + RUN" establishes. As in Volume 2, we need to index atomic wffs to have propositions. For example, we can write:

$$(\text{DOG} + \text{SLEEP})_1 \wedge_{\text{eb}} (\text{DOG} + \text{RUN})_4$$

$$(\text{DOG} + \text{RUN})_4 \wedge_{\text{eb}} (\text{DOG} + \text{SLEEP})_2$$

The indices do not mark specific times. We use them only to distinguish two uses of a single sentence-type as distinct propositions.

It might seems that with these indices we've introduced a notion of thing, for they are numbers. But using numbers is only a convenience for us speakers of English. We could indicate difference or sameness of the use of the type "DOG" by using different fonts, as in **dog**, Dog, DOg, *dog*, *DOG*, DOG, doG, . . . , or by using different colors for the letters. Those would not introduce any thing conception. But I'll use numbers for the sake of simplicity for us English speakers.

Allowing names as base categorematic words, as discussed at the end of Chapter 5, we can see a clearer parallel with what we did in Volume 2. There we considered:

Spot barked. Then Dick yelled. Then Spot barked.

We formalized this as:

$$((\text{Spot barked})_1 \wedge_{\text{eb}} (\text{Dick yelled})_4) \wedge ((\text{Dick yelled})_4 \wedge_{\text{ee}} (\text{Spot barked})_2)$$

In our mass-process language we can write:

$$[(\text{SPOT} + \text{BARK})_1 \wedge_{\text{eb}} (\text{DICK} + \text{YELL})_4] \wedge [(\text{DICK} + \text{YELL})_4 \wedge_{\text{ee}} (\text{SPOT} + \text{BARK})_2]$$

Now a categorematic word by itself can be a proposition, if it is indexed. For example:

$$(\text{CAT})_{16}$$

$$(\text{RUN})_{1947}$$

$(\text{VIRTUE})_{17}$

$(\text{EAT} + \text{MEAT})_{11}$

$(\text{DOG} / \text{FAKE})_{4318}$

$(\text{JUSTICE} + \text{BARK})_{82}$

$(\text{CAT} + \text{LOVE} + \text{HUMAN})_{9,000,001}$

I can assert that there is dog-ing together with sleeping then-there, and I can formalize that with "$(\text{DOG} + \text{SLEEP})_2$". Again, I said that I'm talking of "then-there". But that's a crutch I've used to get you to have some idea of what we're doing here. There is no talk of location, of space; we do not have a logic of spatial connectives, and that's a big lack. We are considering only time. And we are not somehow indicating that I'm talking of now or that you are talking of yesterday. We establish—not pick out—times with true atomic propositions.

27 Subordination in Time

If a subordination such as "JUSTICE sub VIRTUE" is true, then in every context in which it is correct to assert "JUSTICE", it is correct to assert "VIRTUE". Here a context is given not to a proposition but by a proposition relative to other propositions via before and after.

So consider the following true proposition:

DOG sub MAMMAL

Suppose we assert "$(\text{DOG})_2$" and it is true. Hence, it establishes a time. So in that context, that is, during all that time, it is correct to assert "MAMMAL". But perhaps no one did assert "MAMMAL" meaning to establish a time that included the time of "$(\text{DOG})_2$". Propositions, remember, are uttered or written; they do not exist in some timeless place outside our language and world. So how can we say that it is correct to assert "MAMMAL" of the time established by "$(\text{DOG})_2$"?

We can't say there is such an assertion, for there might not be one. Nor can we say that if there is one, it is true, for to say that there is one whose time includes that of "$(\text{DOG})_2$" requires the assertion of "MAMMAL" to be true. What we can say is that any true indexed version of "MAMMAL", say "$(\text{MAMMAL})_3$", either establishes a time that is completely disjoint from that of "$(\text{DOG})_2$" or else if it overlaps that of "$(\text{DOG})_2$" then it includes all of the time of "$(\text{DOG})_2$". That is, "$(\text{MAMMAL})_3$" can't establish a time that covers just part of the time established by "$(\text{DOG})_2$", for directly outside the time established by "$(\text{MAMMAL})_3$", "MAMMAL" is not a correct description.

To formalize this observation, we can use two defined connectives from the logic of temporal connectives, where p and q stand for any indexed mass-process words:

$$p \wedge_x q \equiv_{\text{Def}} [(p \wedge_{bb} q) \wedge (p \wedge_{be} q)] \vee [(q \wedge_{bb} p) \wedge (q \wedge_{be} p)] \vee (p \approx_{bb} q) \vee (p \approx_{ee} q) \vee (p \approx_{be} q) \vee (p \approx_{eb} q)$$

This is true iff p is true and q is true and the time that p establishes intersects the time that q establishes.

$$p \wedge_w q \equiv_{\text{Def}} (p \wedge q) \wedge \neg (p \wedge_{bb} q) \wedge \neg (q \wedge_{ee} p)$$

This is true in a model iff p is true and q is true, and the time that p establishes is within the time that q establishes.

Then we can formalize our observation:

$$[((\text{DOG sub MAMMAL}) \wedge ((\text{DOG})_2 \wedge_x (\text{MAMMAL})_3) \rightarrow ((\text{DOG})_2 \wedge_w (\text{MAMMAL})_3)]$$

The same hold for any indices:

$$[\ (\text{(DOG sub MAMMAL)} \wedge\ (\text{(DOG)}_i \wedge_{\text{x}} \text{(MAMMAL)}_j\ )\ )$$
$$\rightarrow\ \text{(DOG)}_i \wedge_{\text{w}} \text{(MAMMAL)}_j\ ]$$

The general principle for subordination and temporal assertions is the following.

Subordination and temporal assertions For all indices *i* and *j*,

$$[\ \text{(E sub F)} \wedge\ (\ \text{(E)}_i \wedge_{\text{x}} \text{(F)}_j\ )\ ] \rightarrow\ (\ \text{(E)}_i \wedge_{\text{w}} \text{(F)}_j)$$

---

28 A Formal Logic

We incorporate the logic of propositional temporal connectives **TC** from Volume 2 into the base logic of categorematic words and subordination, **MPSub**. As in **TC**, temporal propositional connectives can join only atomic propositions. But now atomic propositions have structure. We invoke that structure in requiring that the temporal connectives can join only indexed categorematic words, not subordination wffs.

Vocabulary
We add to the vocabulary of **MPSub**:

indices 1, 2, 3, . . .

temporal connectives $\wedge_{bb}$ $\wedge_{ee}$ $\wedge_{be}$ $\wedge_{eb}$

Categorematic words
These are defined as for **MPSub**.

Wffs
To the definition of well-formed-formulas of **MPSub**, we add:

• If E is a categorematic word, then for every index *i*, $(E)_i$ is a wff.
It is an *indexed atomic wff*.

• If A and B are indexed atomic wffs, then each of the following is a wff of length 2:

$(A\wedge_{bb} B)$ $(A\wedge_{ee} B)$ $(A\wedge_{be} B)$ $(A\wedge_{eb} B)$

Defined connectives from **TC** are now for only indexed atomic wffs. In particular, we have $p \wedge_x q$ and $p \wedge_w q$ defined when p and q are indexed categorematic words.

Realizations and semi-formal languages
The definition is as for **MPSub**.

Models
We take the notion of a model for **TC**, classical propositional logic with temporal connectives, as defined in Chapter 51 of Volume 2, pp. 205–206. In those models,

$\upsilon(p \wedge_w q) = \top$ iff $\upsilon(p) = \top$ and $\upsilon(q) = \top$ and $\tau(p) \subseteq \tau(q)$

$\upsilon(p \wedge_x q) = \top$ iff $\upsilon(p) = \top$ and $\upsilon(q) = \top$ and $\tau(p)$ intersects $\tau(q)$

We add the following condition.

Subordinations and temporal propositions

If $\vee$ (E sub F) = $\top$, and $\vee$ ($(E)_i \wedge_x (F)_j$) = $\top$, then $\vee$ ($(E)_i \wedge_w (F)_j$) = $\top$.

Axioms

We add to the axiom schemes of **MPSub** the temporal axioms of **TC** from Chapter 54 of Volume 2, pp. 223–224 and an additional axiom scheme for our new conditions on models.

Subordination and temporal assertions

$[(E \text{ sub } F) \wedge ((E)_i \wedge_x (F)_j)] \rightarrow ((E)_i \wedge_w (F)_j)$

We call this logic **MPSub+TC**.

29 Examples of Formalizing

As before, I have no choice but to give examples in English, either as propositions which we can try to re-interpret or as descriptions of an experience we might have that we can try to cast in the view of the world as flux.

Example 1 It's raining.

$(\text{RAIN})_3$

Analysis This seems very unsatisfying. It seems that all we're doing is uttering the word "RAIN". There is no time involved.

But that's because we are looking at this proposition in isolation. Time is not picked out with it, but only established relative to other propositions, like "$(\text{ZOE + SHOP})_{13}$" and "$(\text{DICK + SLEEP})_{102}$".

Still, there is the tense in the example. It's meant to talk about now. All our descriptions in English come with indications of past, present, or future. As in *Time and Space in Formal Logic*, we can formalize talk that uses the past-present-future conception of time by standing outside the system and designating a true indexed atomic proposition or conjunction of true indexed atomic propositions N as establishing now. Here let's take N to be a single atomic proposition. Then for any indexed atomic proposition p, we can define:

Past (p) $\equiv_{\text{Def}}$ $p \wedge_{eb} N$

Future (p) $\equiv_{\text{Def}}$ $N \wedge_{eb} p$

Present (p) $\equiv_{\text{Def}}$ $(p \wedge N) \wedge \neg (p \wedge_{bb} N) \wedge \neg (N \wedge_{ee} p)$

The truth-conditions for these are:

"Past (p)" is true iff p is true and the time of p is before the time of N.

"Future (p)" is true iff p is true and the time of N is before the time of p.

"Present (p)" is true iff p is true and the time of p is within the time of N.

In using our past-present-future talk we assume that time is linear. So in formalizing examples that involve tenses, we'll work in **TCL**, which is **TC** modified to require that the times established by true atomic propositions create a linear ordering, except for those that overlap or are within one another.

So now we can formalize the example as:

Present ($(\text{RAIN})_3$)

Example 2 Whatever is a dog is a mammal

Birta is a dog.

Therefore, *Birta is a mammal.*

DOG sub MAMMAL
BIRTA sub DOG

BIRTA sub MAMMAL

Analysis The informal inference is valid, and the formalization is valid by transitivity of subordination. This is to take the reading of "is" in the example as atemporal subordination.

Example 3 Birta is a dog

Analysis If by this is meant that being a dog is in the essential nature of Birta, we can use "BIRTA sub DOG" as in the last example.

If what is meant is that Birta is a dog right now, though she might have been a fish before or a human in time to come, we cannot formalize the example. That's for the same reason we cannot formalize it in the mass-process logic of times and locations (Example 1, Chapter 14): we cannot disentangle dog-ing from all other mass-process descriptions.

Example 4 Dick yelled before Spot barked.

$(\text{DICK} + \text{YELL})_1 \wedge_{eb} (\text{SPOT} + \text{BARK})_3$

Analysis Some sentences that we understand as predications in English can be formalized in this logic, as they could in the mass-process logic of times and locations (Chapter 14).

Example 5 Dick yelled at the same time as Spot barked.

$(\text{DICK} + \text{YELL})_1 \approx_T (\text{SPOT} + \text{BARK})_3$

Analysis I take the example to be that someone is contradicting the last example, so we should use the same indices as there.

In **TCL** we defined the connective $\approx_T$ such that $p \approx_T q$ is true iff the time interval established by p is the same as that established by q.

We can use the other defined connectives from **TCL** to formalize "while", "during", "within the time", and more from Chapter 53 of Volume 2.

Example 6 Sometime after Dick stopped eating, Spot began to bark.

$(\text{DICK} + \text{EAT})_1 \wedge_{eb} (\text{SPOT} + \text{BARK})_3$

Analysis Despite the apparent quantification over times in the example, we can formalize it (compare Example 4, Chapter 53 of *Time and Space in Formal Logic*).

Example 7 Puff is running from Spot.

Present [$(\text{PUFF} + \text{RUN})_1$ <u>from</u> $(\text{SPOT} + \text{BARK})_3$]

Analysis We can introduce categorematic propositional connectives into this logic as we did in the mass-process logic of times and locations (Chapter 13).

Example 8 Zoe is running from something.

Analysis We have the same problems formalizing this as in the mass-process logic of times and locations (Example 5, Chapter 19). Should we introduce a unary propositional connective "from" or use "from" as a modifier?

Example 9 The dog that barked before is barking again now.

Analysis If we develop a method of talking of the same analogous to what we did in the mass-process logic of times and locations (Chapter 19), we could formalize the example as:

$$\text{Past}\,(\,(\text{DOG})_{13}\,)\,\wedge$$

$$\text{Past}\,(\,([\![\ \text{this}\ (\text{DOG})_{13}\ ]\!] + \text{BARK})_1\,)\ \wedge\ \text{Present}\,(\,([\![\ \text{this}\ (\text{DOG})_{13}\ ]\!] + \text{BARK})_2\,)$$

Example 10 There is mud in the patio now.

$$\text{Present}\,(\,(\text{MUD})_2\ \underline{\text{in}}\ (\text{PATIO})_4\,)$$

Analysis This formalization requires introducing categorematic connectives. But more, it requires treating a location-name as a mass-process word. There, patio-ing. Or there, corral-ing. This is a way to deal with locations that is compatible with the view of the world as process. A location is mass-process, some of the mass-process of space.

We can do this only if we have a way to describe the location. How could we formalize "There is mud somewhere now"? If we assume that all our propositions are of both space and time, we could use:

$$\text{Present}\,(\,(\text{MUD})_5\,)$$

But that assumption would have to stand outside the formal system.

Example 11 There are two patches of mud now in the patio.

$$\text{Present}\,(\,(\text{MUD})_2\ \underline{\text{in}}\ (\text{PATIO})_4\,)\,\wedge$$

$$\text{Present}\,(\,(\text{MUD})_3\ \underline{\text{in}}\ (\text{PATIO})_4\,)\,\wedge$$

$$\neg\,(\,[\![\ \text{this}\ (\text{MUD})_2\ ]\!] \approx [\![\ \text{this}\ (\text{MUD})_3\ ]\!]\,)$$

Analysis This shows that we can count without treating times and locations as things we quantify over (Example 19, Chapter 24). The counting is done by using distinct categorematic expressions that do not describe the same. That we distinguish between different utterances or inscriptions leads to a way that we, as thing-talkers, see as counting.

But talking generally of things and mass-process together with this logic of temporal connectives encounters the same problems we saw with talking of things

and mass-process together with the logic of times and locations for mass-process (Chapter 22).

Example 12 There is no cat-ing in the patio now.

Analysis It seems that we could formalize this with:

$\neg$ Present ($(\text{CAT})_2$ $\underline{\text{in}}$ $(\text{PATIO})_4$)

But that and the following could both be true:

Present ($(\text{CAT})_7$ $\underline{\text{in}}$ $(\text{PATIO})_4$)

We need to say that there is no index *i* such that "Present ($(\text{CAT})_i$ $\underline{\text{in}}$ $(\text{PATIO})_4$)" is true. We did that in *Time and Space in Formal Logic* by introducing quantification over indices, and we could do the same here, formalizing the example as :

$\neg \exists i$ Present ($(\text{CAT})_i$ $\underline{\text{in}}$ $(\text{PATIO})_4$)

But this doesn't actually say that there is no cat-ing now, only that no indexed version of "CAT" is true of now. That is, given the resources of our language, there is no way to say that there is cat-ing now.

This problem arises because only true atomic propositions establish times. We have no atomic proposition that is meant to describe not-cating. Perhaps we could introduce a word negation to accomplish that. We'd write "~CAT" to mean the absence of cat-ing. It would be a mass-process word. Then we could formalize the example as:

Present ($(\sim \text{CAT})_8$ $\underline{\text{in}}$ $(\text{PATIO})_4$)

And we could assert:

$(\text{DOG} + \text{SLEEP} + \sim\text{CAT})_1 \wedge_{\text{eb}} (\text{DOG} + \text{RUN} + \text{CAT})_4$

To do this, we'd need to agree that "~CAT" elicits a concept: the complete absence of cat-ing. So it would figure in subordinations. For example, as a consequence of "CAT sub ANIMAL", we'd have "~ANIMAL sub ~CAT": the complete absence of animal-ing is included in the concept of the complete absence of cat-ing.

What non-negated mass-process concepts would "~CAT" be subordinate to? We can't have "DOG sub ~CAT" because we can have contexts in which there is both dog-ing and cat-ing, so that the principle of subordination and truth in a context would fail. No positive mass-process description being true precludes cat-ing. There are no contrary mass-process words, as noted at the end of Chapter 14.

But surely we should have:

BLACK sub ~WHITE

Conceptually, the concept of black is included in that of not-white. It seems that this is a codification of a correct use of words, too. If so, in every context in which it is

correct to assert "BLACK" it is correct to assert "~WHITE". But then there would be contexts in which it is correct to assert both "WHITE" and "~WHITE", for example if there is (described in thing-talk) a black dog and a white cat in the patio. Thinking in terms of times and locations, in some part of that location at that time there is white-ing, and in some part of the location at that time there is not-white-ing. But that's not what we have with "~WHITE". We're asserting the complete absence of white-ing in that time and location. It's a some versus all issue. For the positive mass-process word to hold it has to be true of some of the location: that's what's codified by the principle of the outward closure of truth for locations. For the negative mass-process word to hold, there has to be complete absence, hence in all of the context the mass-process word does not apply.

The alternative is to treat "~WHITE" as we do "DOG": it's true of a context iff it is true in some part of that context.

There is so much more we could do. But not by me, not now.

Appendices

Appendix 1 **Compound Nouns and Meaning**

If we look to linguists to understand "doghouse", they'll tell us that it means something like "house for a dog". If we ask them how to understand "cartoon cat" they'll tell us it means something like "cartoon of a cat". There are two problems with this.

First, we want to understand the role that "dog" plays in "doghouse" and the role "cat" plays in "cartoon cat". We don't want a paraphrase or an equivalent phrase. Perhaps the best we can say is that "dog" acts like an adjective, and "cat" acts like an adjective. But that's wrong. The word "dog" doesn't act like an adjective; it is an adjective in "doghouse", just as "cat" is in "cartoon cat". We don't have a good explanation of how adjectives mean, how they combine with nouns, nor how adverbs mean, how they combine with verbs. But whatever explanation we do have is exactly what we have for "dog" in "doghouse" and "cat" in "cartoon cat". Linguists give us a list of possibilities for how to understand the link between the two nouns: it could be "for", or "of", or many others as you can see in the extract below. That list explains and clarifies nothing, for we need to know in advance which of the categories applies for the noun-noun combination, and we can determine that only if we understand the combination. The list does not explain our understanding but only gives us choices for describing our understanding.

The second problem is that giving a paraphrase, or a word or phrase that we can substitute for the phrase, or a definition does not give the meaning of a word. I've talked about this in "Language-Thought-Meaning". Now I am reminded of a time in 1976 or 1977 when I was at the logic seminar at Victoria University of Wellington. A philosopher was talking about meaning. He was going to explain the meaning of the word "rabbit". So he wrote on the board:

(a) "Rabbit" means rabbit.

I couldn't understand. I thought he was going to pull a rabbit from his hat to show us, pointing to it and saying "rabbit". I told him that (a) couldn't be right. But, he said, it's just like with:

(b) "Rabbit" means lapin.

(The word "lapin" is the usual translation of the word "rabbit" into French.). I still objected, and he and the others told me I just didn't understand the subject. I was new to philosophy.

But now that I've thought about it more, I'm sure I was right. Both (a) and (b) involve use-mention confusions. Instead of (b), he should have written:

(c) "Rabbit" means the same as "lapin".

And likewise (a) should be:

"Rabbit" means the same as "rabbit".

This last gets us nowhere. At least (c) can be used to help us translate.

But we can't carry around rabbits, and dogs, and unicorns to illustrate what we mean. So we give paraphrases, definitions, equivalent phrases to help someone understand what we're saying. Those are aids to help us learn how to use a word or phrase—if we understand the paraphrase or definition. But that route is of no use at all in explaining the role of a

noun used as an adjective. Here: pick which one of these linking phrases that seems most appropriate for this compound. What we need to say about the role of a noun used as an adjective is that it means there is a link, the noun used as a modifier is meant to link the concept of that noun to the referential use of the noun being modified. We can make a list of possibilities for that link, but that will always be incomplete, as linguists have found in expanding the list again and again. The idea of a link need not be revised. And it is the same with an adjective modifying a noun: it is the concept of the adjective linked to the referential use of the noun.

This is much clearer in talking and reasoning about the world as process, for we embrace rather than notice as a curious aspect of language that any meaningful word is a concept word, and all concept words can play the same roles. There are no verbs, no nouns, no adjectives, no adverbs, but only concept words. Rather than make a list like "dog" (noun), "to dog" (verb), "doggy" (adjective), "doggedly" (adverb), we have the one word "DOG". And then we can have "HOUSE / DOG" for a word that is true in a context if and only if the word "HOUSE" is true and the word "DOG" links that word to the concept of "DOG" in our understanding to apply in that context, where the exact idea of the link depends on our understanding of those two words and, perhaps, the context, an understanding we improve as we use the word more.

* * * * * * * * * * *

Here is an extract from a paper which shows that the description I give of how linguists analyse noun compounds is not a strawman. It is from "On the Semantics of Noun Compounds" by Roxana Girju, Dan Moldovan, Marta Tatu, and Daniel Antohe.

> The semantic interpretation of noun compounds (NCs) deal with the detection and semantic classification of the relations between noun constituents. The problem is complex and has been studied intensively in linguistics, psycho-linguistics, philosophy, and computational linguistics for a long time. There are several reasons that make this task difficult. (a) NCs have implicit semantic relations: for example, "*spoon handle*" encodes a PART-WHOLE relation. (b) NCs' interpretation is knowledge intensive and can be idiosyncratic. For example, to correctly interpret "*GM car*" one has to know that GM is a car-producing company. (c) There can be more than one semantic relation encapsulated in a pair of nouns. For example, "*Texas city*" can be tagged as a PART-WHOLE relation as well as a LOCATION relation. (d) The interpretation of NCs can be highly context-dependent. For example, "*apple juice seat*" can be defined as "seat with apple juice on the table on front of it" (cf. Downing, 1977).
>
> Although researchers (Jespersen, 1954; Downing, 1977) argued that noun compounds encode an infinite set of semantic relations, many agree (Levi, 1978; Finin, 1980) there is a limited number of relations that occur with high frequency in noun compounds. However, the number and the level of abstraction of these frequently used semantic categories are not agreed upon. They can vary from a few prepositional paraphrases (Lauer, 1995) to hundreds and even thousands more specific semantic relations (Finin, 1980). The more abstract the categories, the more noun compounds are covered, but also the more room for variation as to which category a compound should be assigned. Lauer (Lauer, 1995), for example, considers eight prepositional paraphrases as semantic classification categories: *of, for, with, in, on, at, about,* and *from*. According to this classification, the noun compound "*bird sanctuary*",

> for instance, can be classified both as "*sanctuary* **of** *bird*" and "*sanctuary* **for** *bird*". The main problem with these abstract categories is that much of the meaning of individual compounds is lost, and sometimes there is no way to decide whether a form is derived from one category or another.
>
> On the other hand, lists of very specific semantic relations are difficult to build as they usually contain a very large number of predicates, such as the list of all possible verbs that can link the noun constituents. Finin (1980), for example, ?? semantic categories such as "**dissolved in**" to build interpretations of compounds like "*salt water*" and "*sugar water*". Although, there were several proposals of possible large sets of semantic relations, there has been no attempt to map one set to another, and, more importantly, to define the appropriate level of abstraction for the interpretation of compounds in general, or for a specific application in particular. pp. 479–480

The authors then proceed to an analysis "using two sets of semantic classification categories: a list of 8 prepositional paraphrases previously proposed by Lauer [reference] and a new set of 35 semantic relations introduced by us."

Here are some problems with this approach beyond what I suggested above.

1. By not seeing the use of a noun in a noun compound as an adjective, it gives no guidance for how to compare, for example, "doggy smell" and "dog smell".

2. The authors do not clarify the relation of noun compounds to noun conjunctions, for example, "dog love" and "dog and love". Indeed, the former is ambiguous: does it mean love for dogs or love by a dog? Yet the latter is clear: there (pointing) is dog and love.

3. The authors do not see that "spoon handle" can be read as not giving a part-whole relation but saying what kind of handle: one for a spoon. That fits into the analysis of modifiers given in Volume 1, *The Internal Structure of Predicates and Names.*

4. The authors do not realize or at least do not say that their work has application to only noun compounds in English. That's because:

   (a) Prepositions are notoriously difficult to translate into other languages, even into other European languages much less into a language that has only one or two prepositions.

   (b) In some mass-process languages, such as Navajo and Wintu (see my essay "Nouns and Verbs"), all the base words can be used in compounds: there simply is no division of words into nouns, adjectives, verbs, and adverbs.

Appendix 2 **Strawson on Mass Terms and Individuals**

In his book *Individuals,* P. F. Strawson discusses mass terms and how to think of them in relation to our talk of individuals. He says:

> I have in mind what I shall call *feature-universals* or *feature-concepts,* and what I shall call *feature-placing statements.* As examples, I suggest the following:
>
> Now it is raining.
> Snow is falling.
> There is coal here.
> There is gold here.
> There is water here.
>
> The universal terms introduced into these propositions do not function as characterizing universals. *Snow, water, coal* and *gold,* for example, are general kinds of stuff, not properties or characteristics of particulars; though *being made of snow* or *being made of gold* are characteristics of particulars. Nor are the universal terms introduced into these propositions sortal universals. No one of them of itself provides a principle of distinguishing, enumerating and reidentifying particulars of a sort. But each can be very easily modified so as to yield several such principles: we can distinguish, count and reidentify *veins* or *grains, lumps* or *dumps* of coal, and *flakes, falls, drifts* or *expanses* of snow. Such phrases as 'lump of coal' or 'fall of snow' introduce sortal universals; but 'coal' and 'snow' *simpliciter* do not. These sentences, then, neither contain any part which introduces a particular, nor any expression used in such a way that its use presupposes the use of expressions to introduce particulars. Of course, when these sentences are used, the combination of the circumstances of their use with the tense of the verb and the demonstrative adverbs, if any, which they contain, yields a statement of the incidence of the universal feature they introduce. For this much at least is essential to any language in which singular empirical statements could be made at all: viz. the introduction of general concepts and the indication of their incidence. But it is an important fact that this can be done by means of statements which neither bring particulars into our discourse nor presuppose other areas of discourse in which particulars are brought in.
>
> Languages imagined on the model of such languages as these are sometimes called 'property-location' languages. But this is an unfortunate name: the universal terms which figure in my examples are not properties; indeed, the idea of a property belongs to a level of complexity which we are trying to get below. This is why I have chosen to use the less philosophically committed word 'feature', and to speak of 'feature-placing' sentences. pp. 202–203

Note that Strawson does not consider beauty or justice. Are these feature-placing? As soon as we say that, we see that Strawson has introduced a major metaphysics here, for he says he is talking about "feature universals" or "feature concepts", where in my work I talk about the words "beauty" and "justice". We can say "Beauty here now" and "Justice there then". We can have "an instance of beauty" and "an instance of justice". These are different classifiers than we use with mass substance terms, for which the standard is "a bit of", as in "a bit of water", "a bit of snow". Perhaps Strawson would say that "justice" and "beauty" are not "really" features, not really masses. But why? Because they have adjective forms?

So does "snow" with "snowy". It has to be, I think, because "justice" and "beauty" aren't substances. But then why is it that we can assert "That is an instance of justice"? Isn't that instance a part of the world? How is it different from objects and masses?[1]

What does Strawson mean by "a statement of the incidence of the universal feature"? I almost agree with him when he says:

> For this much a least is essential to any language in which singular empirical statements could be made at all: viz. the introduction of general concepts and the indication of their incidence. p. 203

This sounds like the categorematic words I discuss. But I don't know what he means by "incidence". Perhaps that is what I call "truth in a context".

Strawson continues:

> Though feature-placing sentences do not introduce particulars into our discourse, they provide a basis for this introduction. The facts they state are presupposed, in the required sense, by the introduction of certain kinds of particular. That there should be facts statable by means of such sentences as 'There is water here', 'It is snowing', is a condition of there being propositions into which particulars are introduced by means of such expressions as 'This pool of water;' 'This fall of snow'. In general, the transition from facts of the presupposed kind to the introduction of the particulars for which they supply the basis involves a conceptual complication: it involves the adoption of criteria of distinctness and, where applicable, criteria of reidentification for particulars of the kind in question, as well as the use of characterizing universals which can be tied to a particular of that kind. A *basis* for criteria of distinctness may indeed already exist at the feature-placing level. For where we can say 'There is snow here' or 'There is gold here', we can also, perhaps, say, 'There is snow (gold) *here*—and *here*—and *here*.' Factors which determine multiplicity of placing may become, when we introduce particulars, criteria for distinguishing one particular from another. pp. 203–204

Chapter 7 of his book is titled "Language without Particulars". This sounds like what we're doing in this book. But he doesn't go with a language without particulars. He focuses on times and places as possible things, and with those as things he says that you can get particulars, which I show in Chapter 25 is not so. He talks about identifying a place by an object that is exactly in it: "Suppose there were a block of granite which maintained its position and its boundaries unchanged." (p. 223) But I showed in Chapter 35 of *Time and Space in Formal Logic* that this a fantasy and not a a method of identification.

If we take Strsawson's just the feature-placing without any direction towards particulars, we have what Han Xiaoqiang describes of Chinese in "Maybe There Are No Subject-Predicate Sentences in Chinese":

> It seems possible, therefore, to hypothesize that the Chinese sentences which are regularly translated as subject-predicate sentences in English are in fact all feature-placing sentences, sentences like "It is snowing here."*
>
> *[footnote] Chad Hansen proposes that (classical) Chinese ontology, which is basically a stuff ontology, an ontology that represents the world as a collection of overlapping and interpenetrating stuffs, goes along with a mass noun syntax that dominates Chinese and especially classical Chinese (Hansen, 1983). Richard Sharvy expressed a similar view with regard to the role of mass nouns in Chinese in his paper (Sharvy 1978: 345–365).

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[1] We can't elude this issue by invoking events as I explain in "Why Event-Talk Is a Problem".

It is perhaps easy to construe all assertive sentences in Chinese as feature-placing sentences, if all Chinese nouns are understood as mass nouns.[2]

[2] Chad Hansen, *Language and Logic in Ancient China.* Richard Sharvy, "Maybe English Has No Count Nouns: Notes on Chinese Semantics. An Essay in Metaphysics and Linguistics", *Studies in Language*, vol. 2, no. 3, 1978, pp. 345–365.

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