Pedagogy and Examples

This section publishes articles that present ideas or examples for the teaching of critical thinking and logic, or examples that either illustrate or call into question some aspect of current theories of the foundations of reasoning.

Knowing the Exact Proportions is Not Enough

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To describe a population we give the percentage of its members that have a certain property. This is a convenient method for dealing with large groups. This is also the way to concentrate on the properties we're interested in. For instance, one may be interested to know how many men and how many women visit a museum in order to know whether the exhibition is equally interesting to both genders. The exact numbers are not that important; we want to know the percentages.

We are all familiar with the need to be very careful when dealing with samples. To infer a property of the whole population, from which the sample is drawn, we must be reasonably sure that the sample is representative and not too small. Elaborate theories of statistics tell us how to choose samples. To know how many visitors to a museum are male we study an appropriate sample and infer the percentage with a probability that can be estimated. It may seem that exact knowledge of the percentage guarantees safe inferences. It turns out that we can err even then. Thus, we may have exact information concerning the ratio of men and women among the visitors, but we can misunderstand the significance of the ratio.

A well known joke explains the danger of statistics: When my neighbor beats his wife every day and I never do, statistically we beat our wives every second day. It could seem that this is so trivial that a little care is sufficient to overcome the danger. Let us see how most of us could fall into the same trap.*

Example 1

In a prestigious college, The School of Marketing and Merchandising, only those candidates are admitted who have passed an entrance examination. A journalist of the local newspaper *Valuable News* became interested in the results of the last exams, and she found out that 36% of women candidates and 64% of men candidates were admitted. "Therefore," she said to herself, "there has been discrimination here!" Things looked obvious, especially in view of the fact that an equal number of men and women took part in the exams, namely, 500 women

* I first learned about an example of this kind from Dr. Lech Kubik, Warsaw.

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(only 180 of whom were admitted) and 500 men (320 admitted). The journalist was ready to write a story about the lack of equal opportunities; she felt that she had enough evidence to make a strong case against the policy of the college. Before writing the final draft she went to interview the president. The president felt embarrassed, and a little threatened, so she decided immediately to check the reason behind the inequality in admissions. There were two separate departments in the college, the Department of Marketing (let us call it Department I), and Department of Merchandising (let's call it Department II), with separate examinations. The president hoped that only one of the departments had been guilty, but the disparity, 36% vs. 64%, was so big that she also felt she had to prepare herself for the possibility that neither of the departments had provided fair examinations. This seemed highly probable because an equal number of candidates, 500, tried their luck in each of the departments. The president called the deans of the departments to report their respective percentages. To her amazement each of them maintained that in his department the chance to be admitted were actually better for women. Both mentioned a 5% margin. Actually, in Department I, from among their candidates, 25% of the women and 20% of the men were admitted. In Department II, 80% of the women and 75% of the men were admitted.

The president didn't believe her deans, and the journalist was sure that they tried to cheat her. "How come," she said, "it isn't obvious that whenever relatively more men than women are admitted, then if not in both departments, at least in one of them it must have been easier for men to be admitted than for women?" The president had no answer to that. She asked the deans to deliver the complete data at once. Only the secretary, who was witnessing the conversation when she brought coffee to the president's room, remarked humbly that if more men were admitted, then more men must have been admitted in at least one of the departments, but perhaps the chance of admission was something else. This looked like casuistry to both her boss and the journalist. The latter said, "If the chance to be admitted is higher for women at both Department I and Department II, and the college has just these two departments, the chance must be higher for all women candidates to the college." "This looks almost like a syllogism to me," admitted the president. Soon, however, they got the Table with all details.

	Department I 500 Candidates		Department II 500 Candidates	
Candidates' gender	400 f	100 m	100 f	400 m
Admitted	100 f	20 m	80 f	300 m
Percentages	25%	20%	80%	75%

	College globally 1000 Candidates	
Candidates' gender	500 f	500 m
Admitted	180 f	320 m
Percentages	36%	64%

The president and the journalist studied the table briefly and did feel humbled, so rather than asking the secretary they brought Dr. E, the school's Critical Thinking instructor, to analyze the situation.

Example 2
Everything is exactly the same. Only when they are waiting for the exact data, another journalist comes. He checked that in both departments women had higher chances to be admitted. He was puzzled because he had always thought that men were much better in marketing and in merchandising. He suspected too much of a political correctness at work, and contemplated an article for the local weekly, *Invaluable News*. When confronted with the table with all the details he felt as bad as did the others.

It was easy for Dr. E to describe the table. Everyone can see that the uneven distribution of men and women among the candidates to the departments is the key fact. Had the distribution been even, the peculiar situation could not have arisen. More women were admitted to Department I, but many more men are admitted to Department II, so in total there are more men. Still, the percentage is always on the side of women because many more women applied to Department I, where only 120 were admitted (while in Department II as many as 380 were admitted). Clearly, women concentrated in the more difficult of the two departments. These remarks of Dr. E seem to have explained the situation well enough.

The journalist (as well as her colleague of Example 2) was, however, unsatisfied. She did not feel she knew what was happening. Was there discrimination or not? If so, in what direction? She did not know how to ask. Fortunately, the secretary helped to ask the right question again: "Why were more women concentrated in the difficult department?"

The question seems hard to answer, and it is surely hopeless if all we know is the Table. Now, one important issue is as follows: What is the reason for the disparity in the number of students admitted (120 to 380)? One possibility (Case 1) is that the number of admissions in each department had been determined in advance. (They were not equal, because, for example, Department I needed special lab classes and could admit no more than 120.) Another possibility (Case 2) is that there was no prearranged numbers, but that they just took all those (top 500) who got good enough grades; it so happened that the candidates to Department II got so much better grades. How does our principal problem, that concerning the gender discrimination, look in both cases?

Case 1. If it had been prearranged that only 120 would be admitted to Department I, why were so many women going there? It was clear that the exams would be more difficult than in the other department. Was it a coincidence? Were women more ambitious? Misinformed? If they chose Department I consciously, there was no discrimination. Were women more attracted because of a specific character of Department I? Did they know that no more than 120 would succeed? If so then, again, there was no discrimination.

It could conceivably have happened that the decision to admit only 120 to Department I was made only after it had been learnt that women were dominating among the candidates to that department. If *that* were the reason for the decision, that is, the fact of women being the majority, then we have a blatant case of discrimination. If, however, the decision was made on other grounds, without taking into account the relative number of women, then there was no discrimination.

Case 2. If the departments admitted just those who got better grades, the problem is why the women came out inferior. We can't exclude the possibility that in that specific place and time men were simply better, on the average. However, before admitting this, we must ask one important question: Were the exams in the two departments of the same degree of difficulty?

If not, was this connected to the fact that there were so many women trying to get to Department I, or not? If yes, then there may have been discrimination; yet, before saying this our journalist should find out the basis of the decision to make the exams to Department I harder. If not, that is, if the exams for Department I were made more difficult, but this had no connection to the relative number of women, then perhaps women just had bad luck (or rather, relatively more women than men turned out to have bad luck). This is rather clear if among those who prepared exams there had been no awareness that women were over-represented among candidates to Department I.

If, on the other hand, the exams were the same all over the college there seems to have been no discrimination. Unless, that is, some subtler reasons intervened.

There may be other factors that could affect the outcome of the exams. For example, the questions could have been posed by a computer, whose software contained an unintended mistake and gave harder questions to those with longer first names. It so happened that women had, on average, longer first names than did men. Then there would be an unfair discrimination, although not intended by anyone, so it would be hard to assess guilt.

We just need to imagine the possibilities. It seems plausible that there can be, theoretically, a large, practically unlimited, number of them.

It should be clear by now that we can easily add additional complexity to the picture. Let us distinguish another category among the candidates, not just according to gender. The new group, call it A, would be, typically, a minority. For example: Athletes, or Asians, or Arabs, or Aristocrats. Or, say, Jews, or Cyclists, or just the opposite. (By the way, we can easily imagine that Asians had, on average, shorter names and then they would be put in a privileged position by that unintended error in software.) Now we are ready for the third example, which is a slight extension of the initial one.

Example 3 Everything is as before, only when they were contemplating the Table and waiting for Dr. E, another journalist came, a columnist for *Valuable News*. She was worried because it has turned out that A-candidates were discriminated against. Namely, only 30% of them were admitted while 50% of all candidates were. And, of course it turned out again that in no department had there been a problem. In Department II there were simply no A-candidates at all. In Department I, 30% of A-candidates were admitted and only 20% of non-A-candidates. The table for Department I was the following ('a' stands for A-ness, 'n' for non-A-ness):

	500 Candidates	
Candidates' A-ness	200 a	300 n
Admitted	60 a	60 n
Percentages	30%	20%

Combined with gender, the table for Department I was:

	500 Candidates						
Candidates' A-ness	400 f =	100 a +	300 n	100 m =	100 a +	0 n
Admitted	100 f =	40 a +	60 n	20 m =	20 a +	0 n
Percentages	25%	40%	20%	20%	20%	

It is also apparent that A-women candidates fared better than the non-A-women. The A-candidates had bad luck to try to get to the department with harder examinations. Or, was it just bad luck? What was the mechanism of the examinations? Were the A-women privileged? If the exam was more difficult for Department I, and within it they were not treated better than others, they were actually the best scoring group of all the specific subgroups of candidates considered so far.

So: Was the A-group at a disadvantage? Or was it the non-A-group? Was any group discriminated against? If so, by whom? Not enough was known to offer the readers of *Valuable News* a reasonable conclusion: The percentages are so-and-so, therefore . . .

Our overall conclusion is that there may be many circumstances, and we must not assume that we know all of them before we analyze the situation. Knowing the exact percentages is not sufficient for drawing conclusions about the reasons for differences or disparities among them. Sounds discouraging? I would certainly grant that in most everyday situations we don't have to be so penetrating. We assume that the factors other than those obviously affecting the situation are irrelevant. But if we want to accuse we should do better. The journalist's task is not just to learn the percentages and infer the existence of discrimination or the lack of it. Having come across the percentages, the journalist still has to do the real work.

Our journalist from *Valuable News* has learnt a valuable lesson. She had a strange dream at night. Various percent figures were dancing and singing to a haunting tune with bizarre words: "It is so hard, so hard for us all, to know the right way of saying 'therefore'." (Once she heard "sixth" rather than "right.")